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WAVE TRANSLATION SYSTEM

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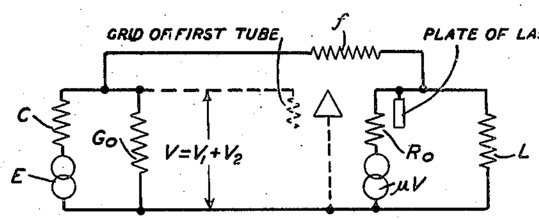
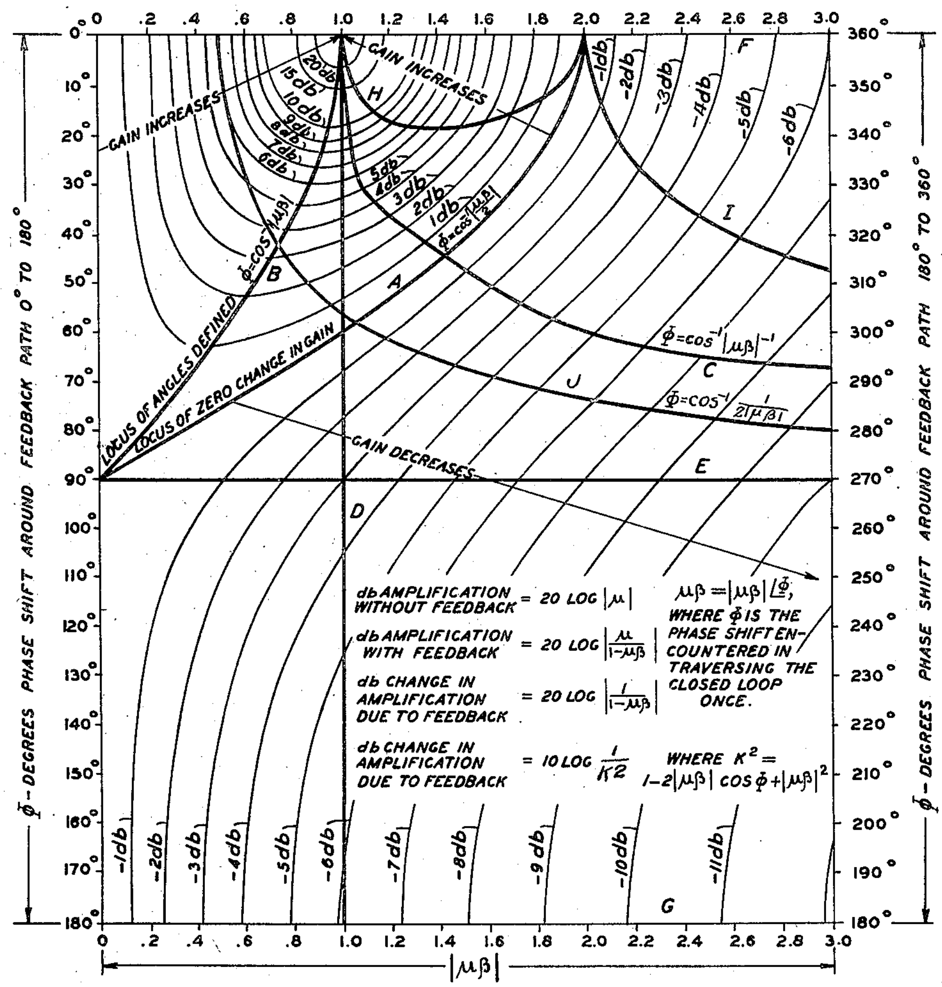


FIG. 1

FIG. 2



μβ, WHICH IS INDICATIVE OF THE AMOUNT OF FEEDBACK, IS A COMPLEX QUANTITY WHICH MAY BE VIEWED AS REPRESENTING THE RATIO BY WHICH THE AMPLIFIER AND FEEDBACK CIRCUIT (OR IN GENERAL μ AND β) MODIFY A VOLTAGE IN A SINGLE TRIP AROUND THE CLOSED PATH.

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FIG. 3

EXTENSION OF FIG. 2 CHART TO LARGER VALUES OF $|\mu\beta|$

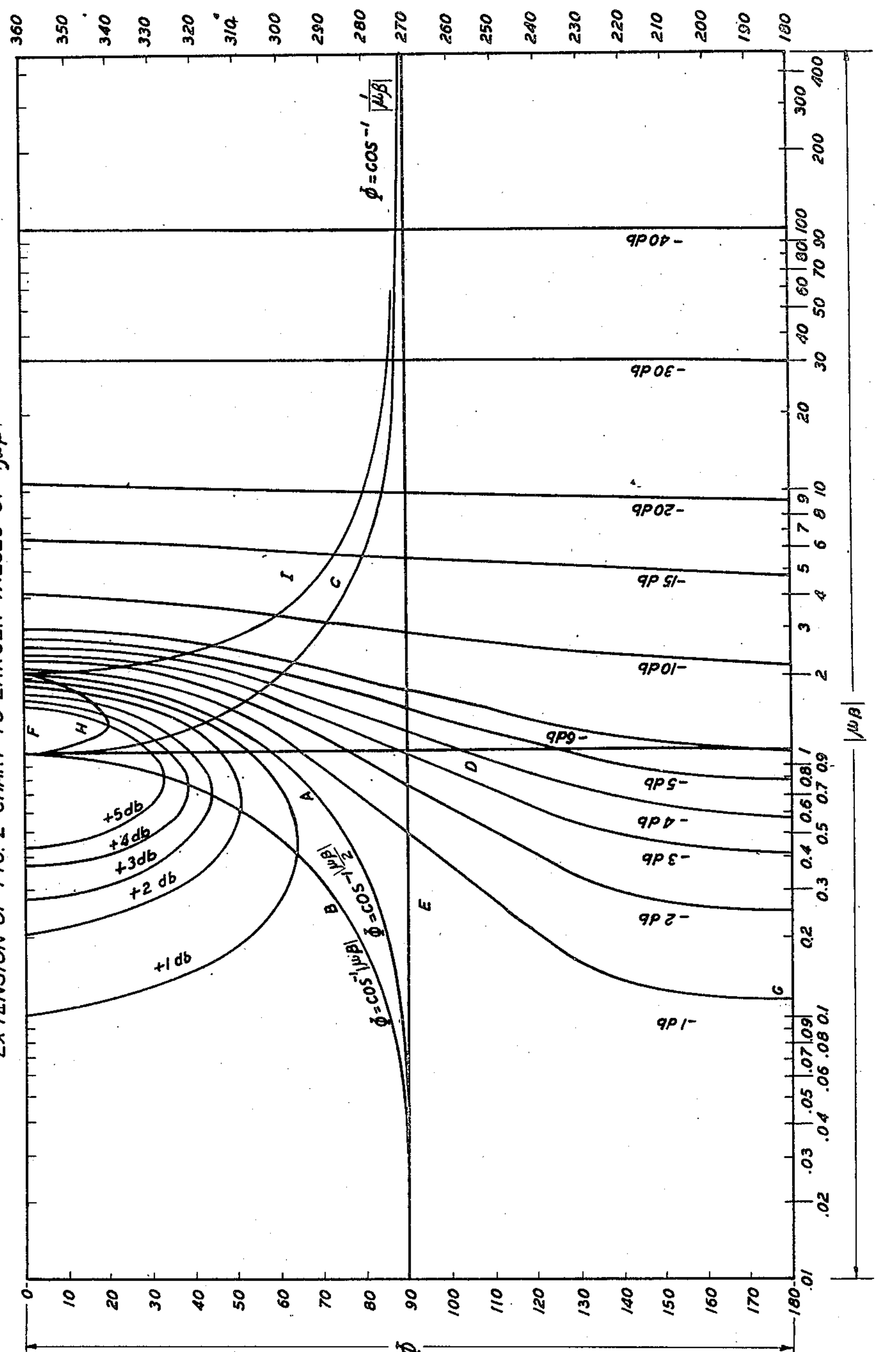

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FIG. 4

POLAR PLOT OF FUNCTIONS PLOTTED IN FIGS. 2 AND 3

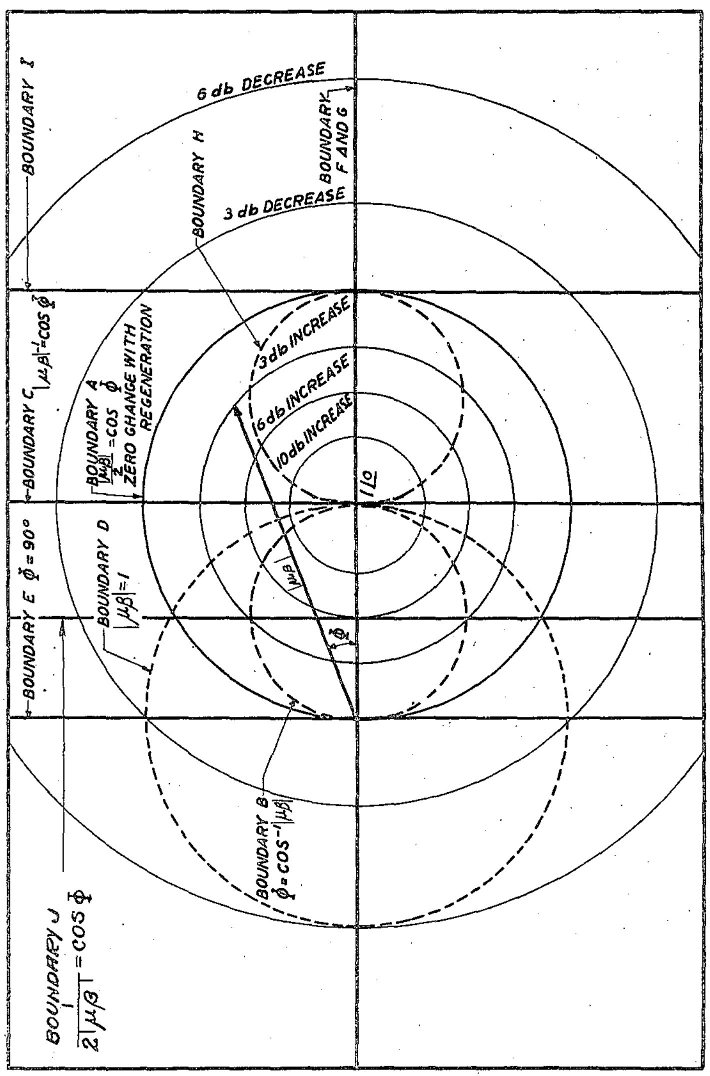

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FIG. 5

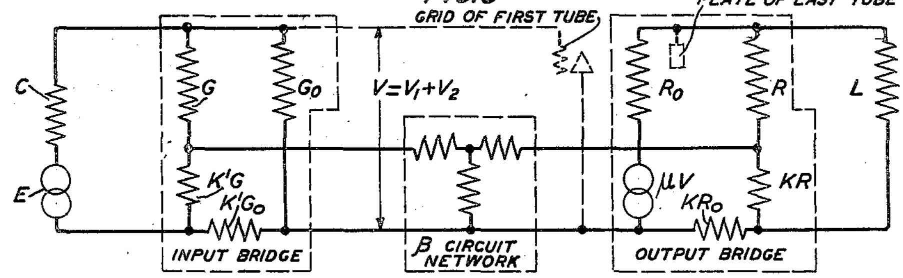


FIG. 6

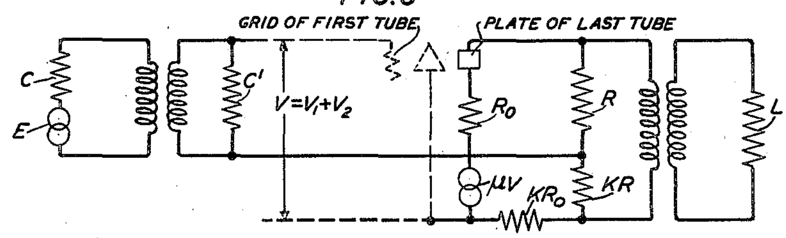


FIG. 7

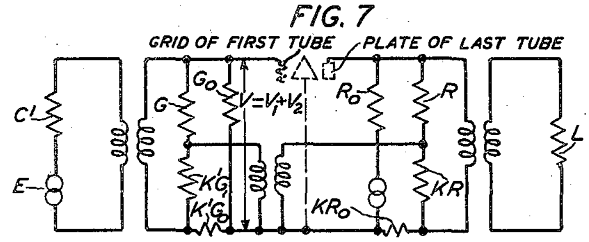
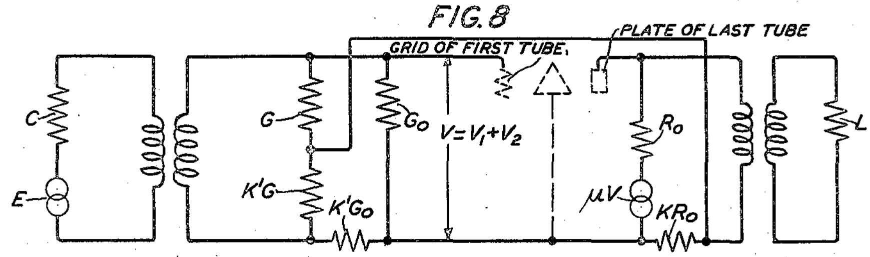


FIG. 8



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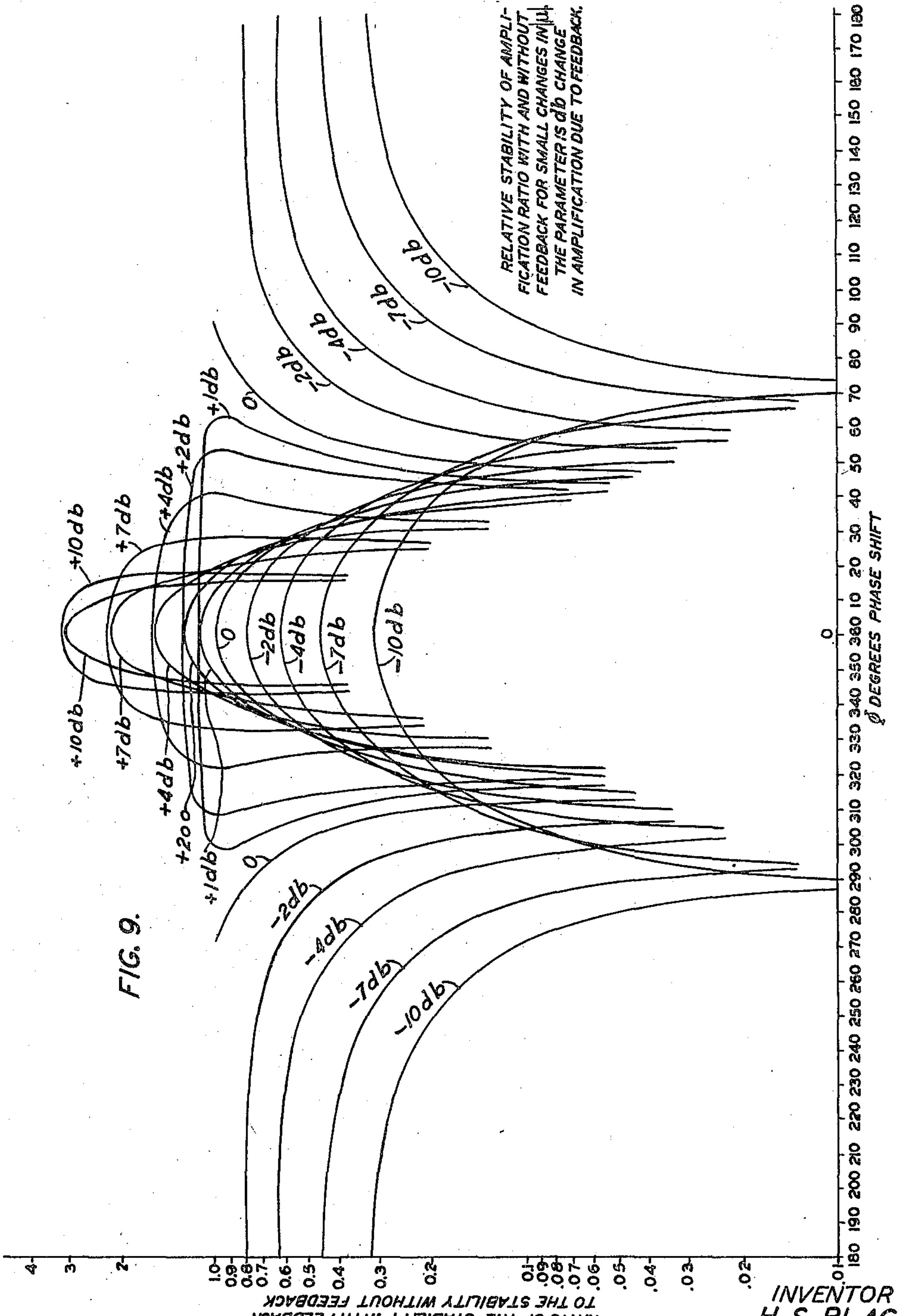

FIG. 9.

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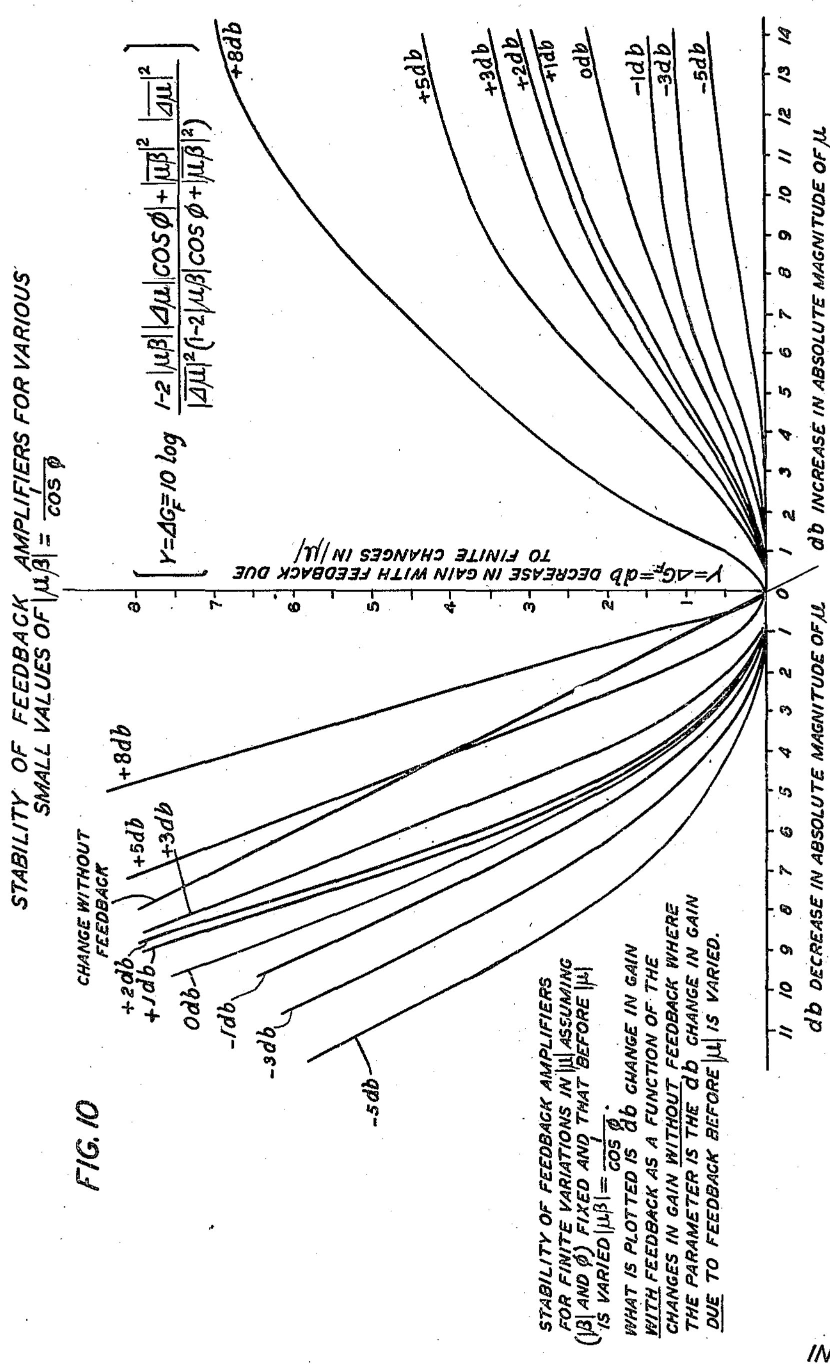

FIG. 10

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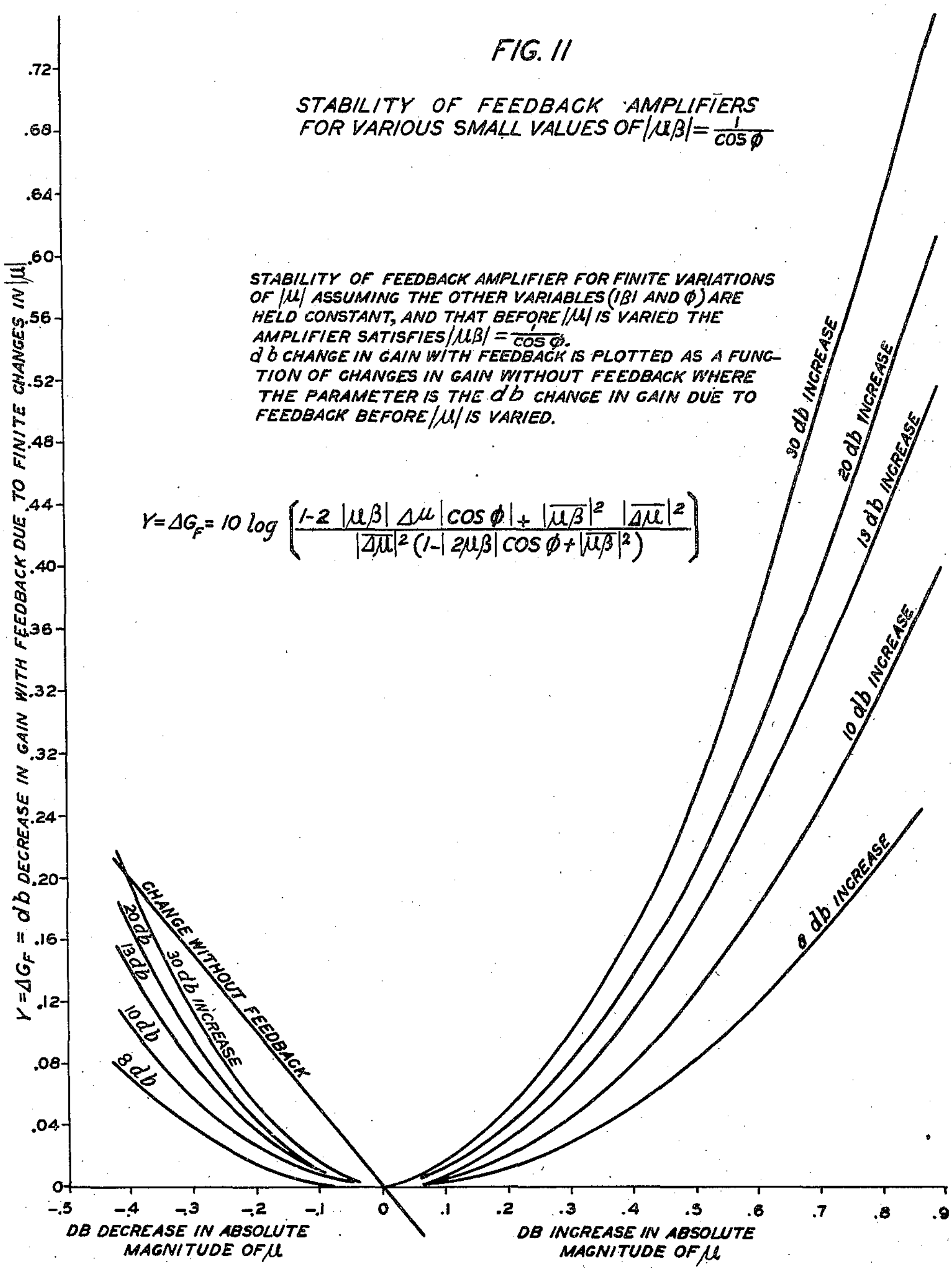

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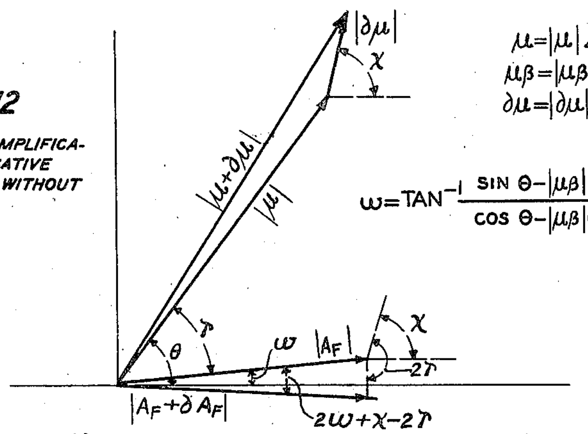
FIG. 12

STABILITY OF AMPLIFICATION WITH NEGATIVE FEEDBACK AND WITHOUT FEEDBACK.



$\mu = |\mu| \angle \theta$

$\mu\beta = |\mu\beta| \angle \phi$

$\partial\mu = |\partial\mu| \angle \chi$

$$\omega = \mathrm{TAN}^{-1} \frac{\sin\theta - |\mu\beta| \sin(\phi-\theta)}{\cos\theta - |\mu\beta| \cos(\phi-\theta)}$$

$$\left(\frac{\partial A_F}{A_F}\right)_\mu = |\partial\mu| \frac{1}{|\mu| \sqrt{1 - 2|\mu\beta|\cos\phi + |\mu\beta|^2}} \angle \chi-\theta-\tau$$

WHERE τ IS THE ANGLE OF $(1-\mu\beta)$

$$\tau = \mathrm{TAN}^{-1} \frac{|\mu\beta| \sin\phi}{1 - |\mu\beta| \cos\phi}$$

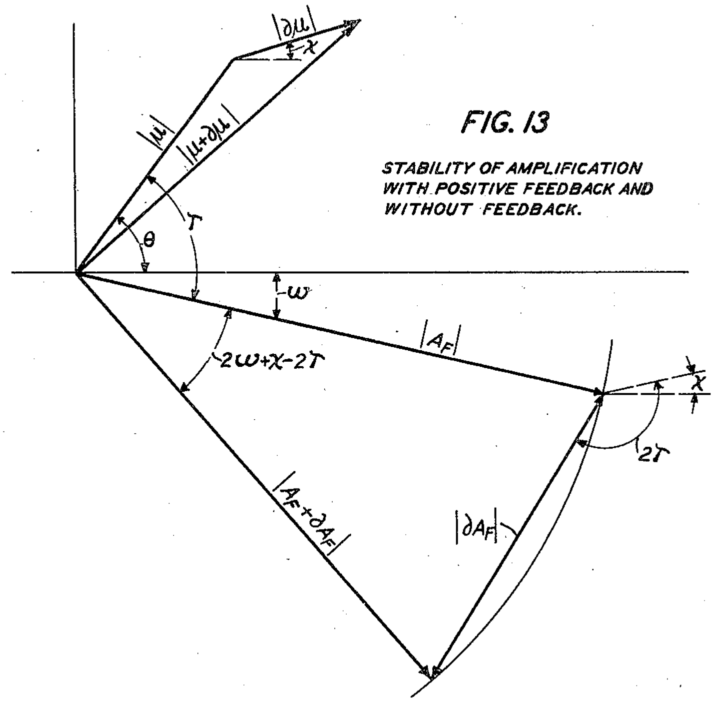


FIG. 13

STABILITY OF AMPLIFICATION WITH POSITIVE FEEDBACK AND WITHOUT FEEDBACK.

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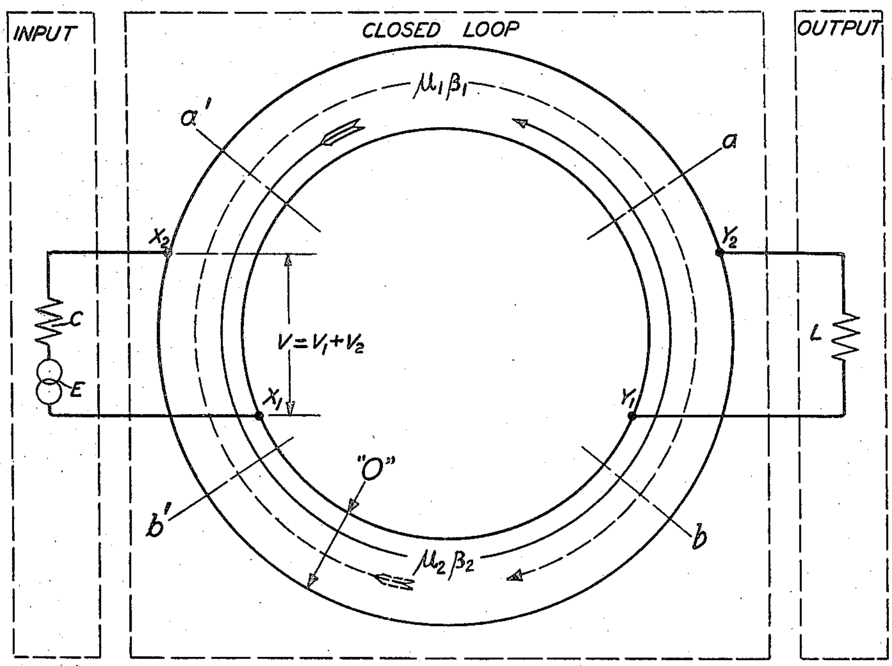
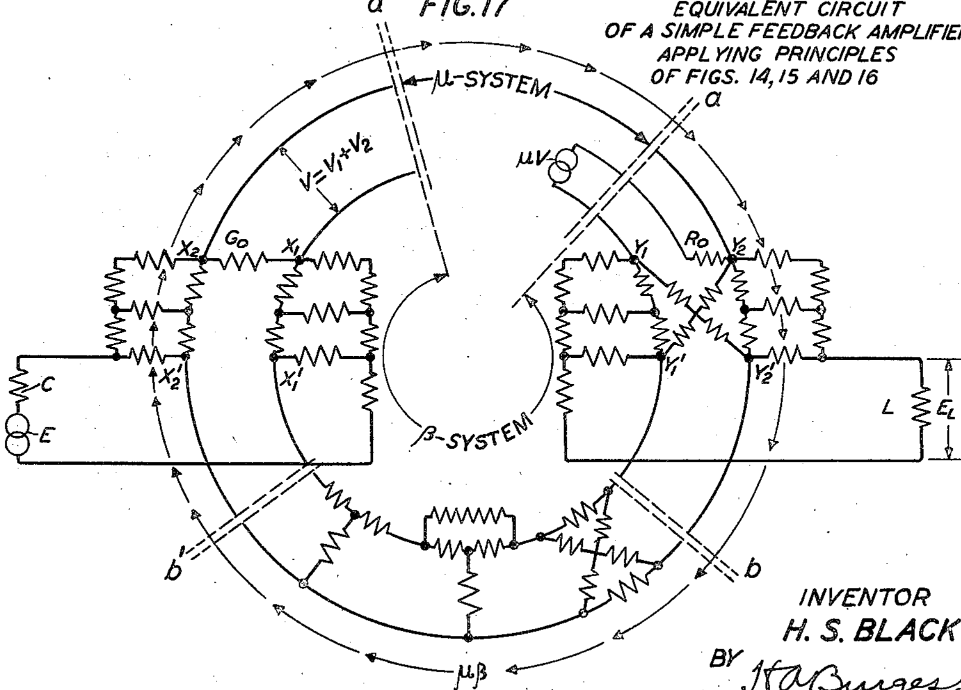
FIG. 14



SYMBOLIC REPRESENTATION USED IN DETERMINING μ AND β

FIG. 17

EQUIVALENT CIRCUIT OF A SIMPLE FEEDBACK AMPLIFIER APPLYING PRINCIPLES OF FIGS. 14, 15 AND 16



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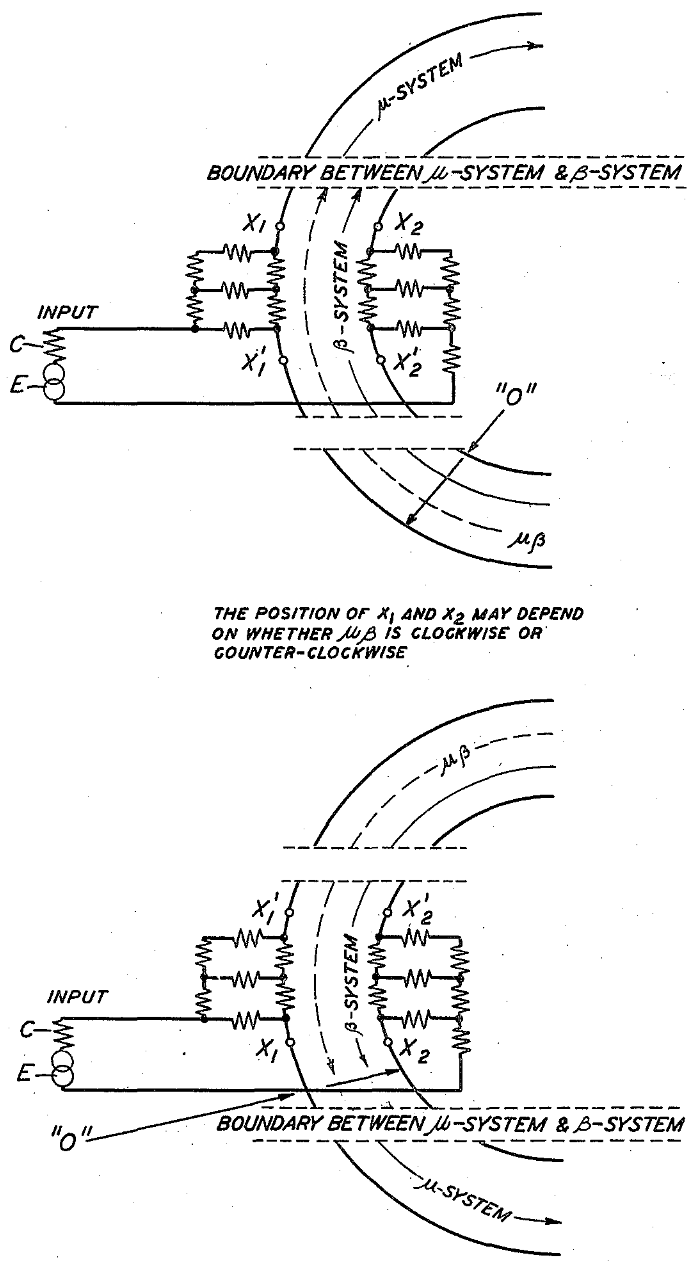

FIG. 15

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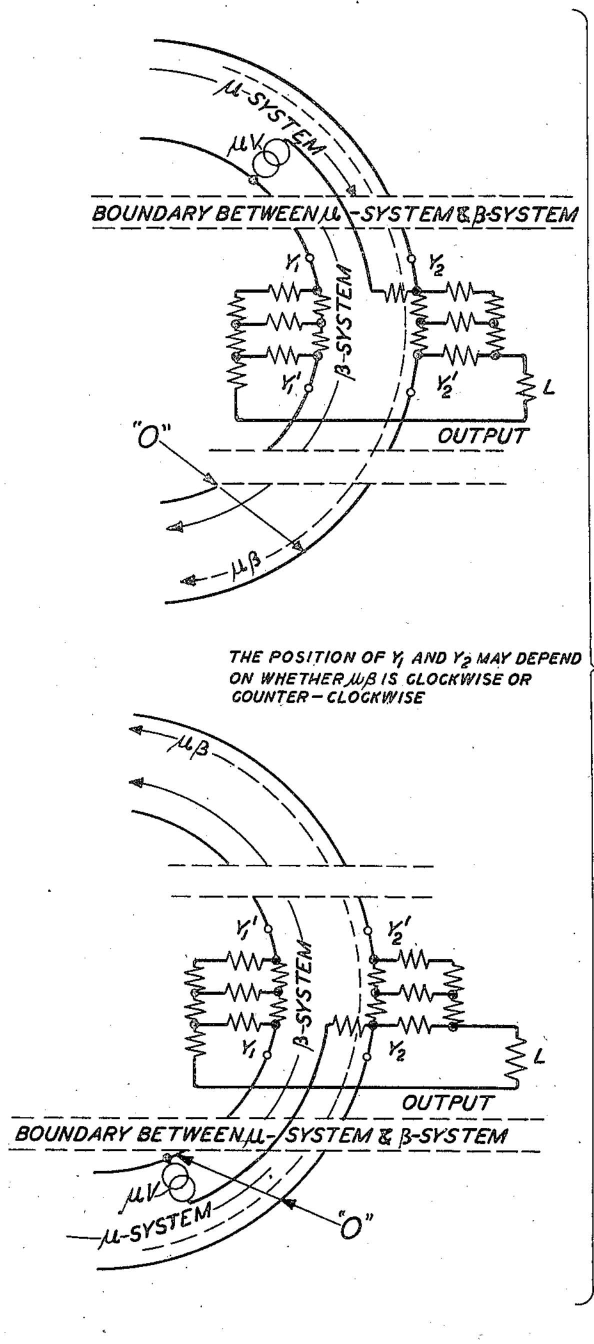

FIG. 16

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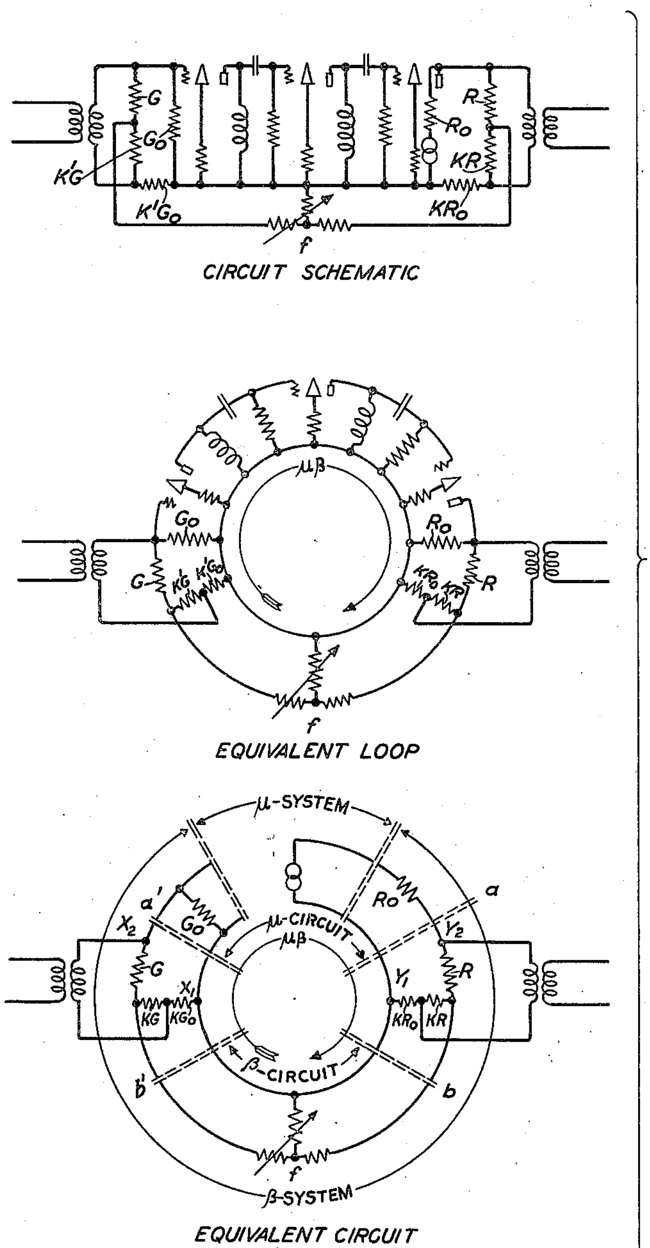

FIG. 18
ILLUSTRATIVE APPLICATION OF METHODS OF FIGS. 14, 15, 16, AND 17

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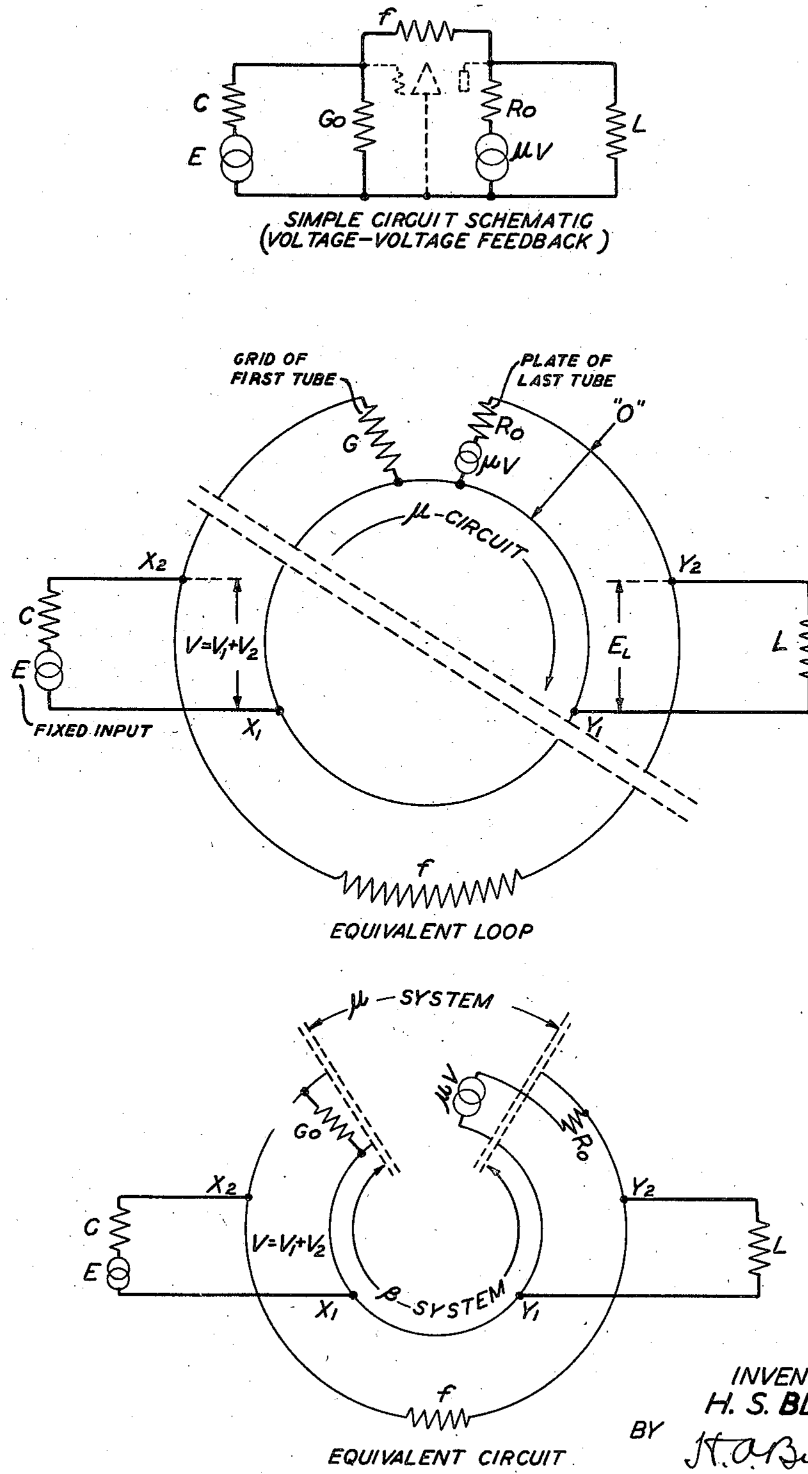

FIG. 19

ILLUSTRATIVE APPLICATION OF METHODS OF FIGS. 14, 15, 16 AND 17

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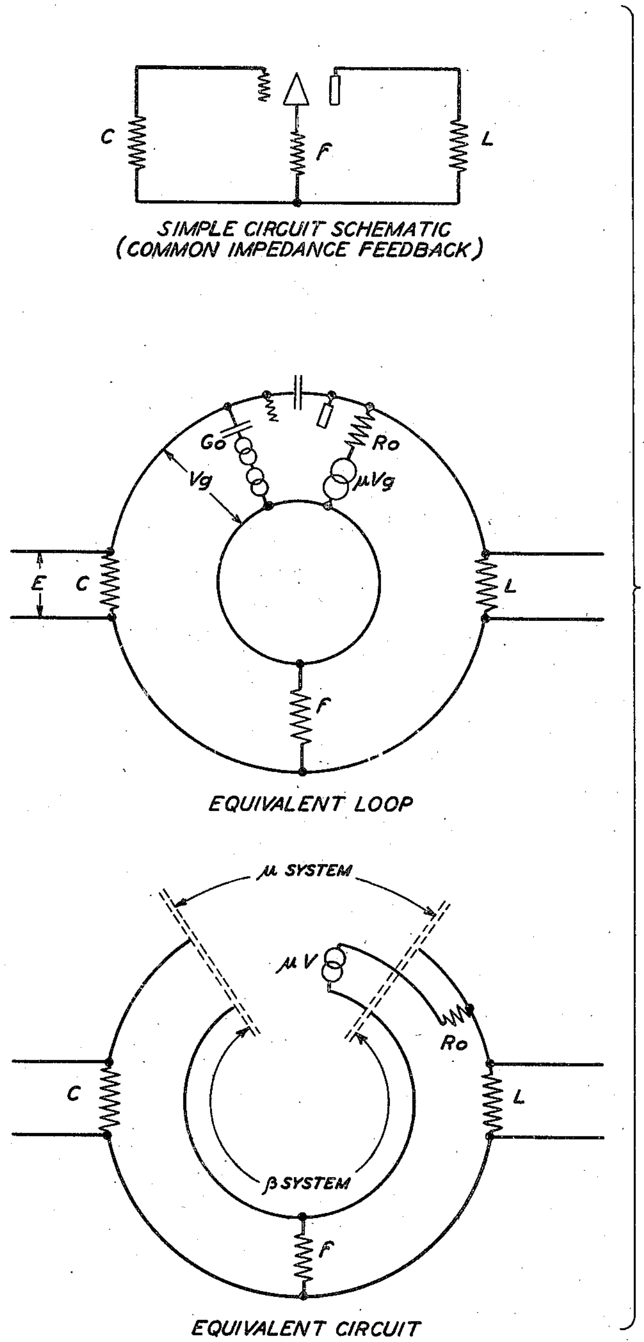

FIG. 20
ILLUSTRATIVE APPLICATION OF METHODS OF FIGS. 14, 15, 16 AND 17

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FIG. 21

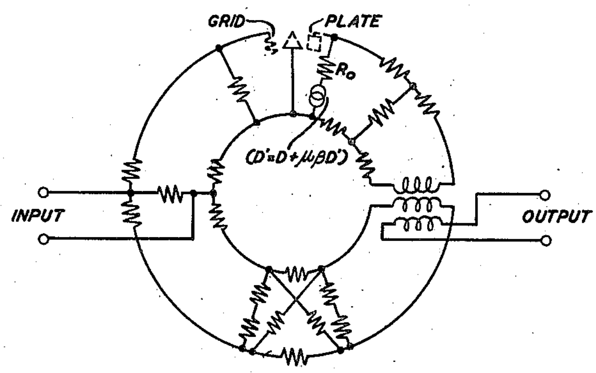

FIG. 22

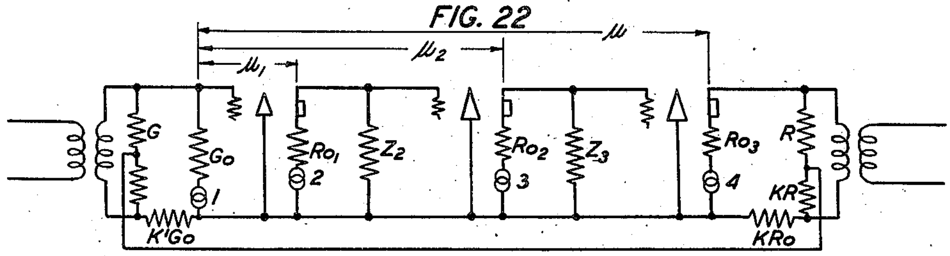

FIG. 23

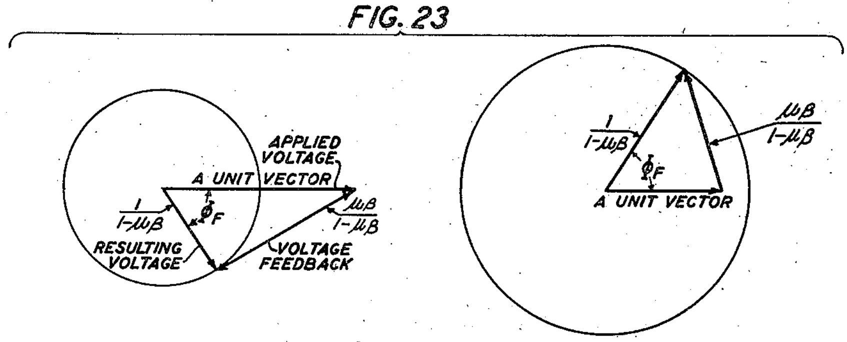

FIG. 24

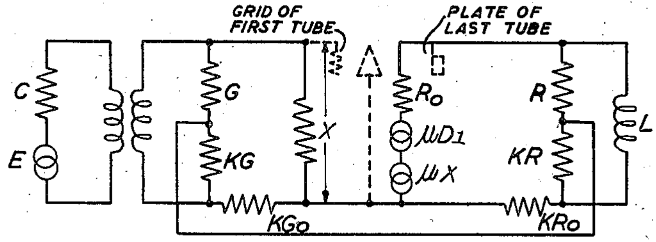

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FIG. 25

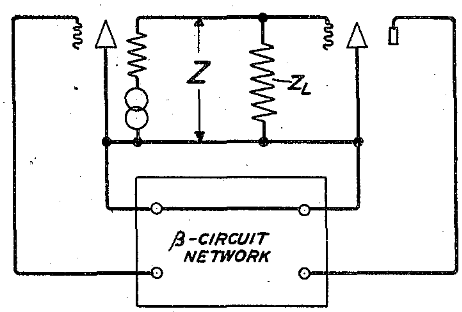


FIG. 26

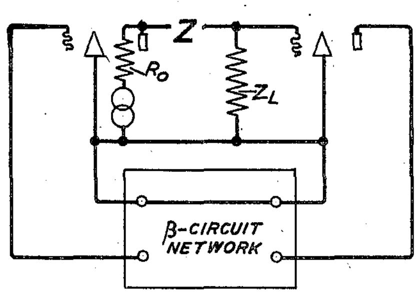


FIG. 27

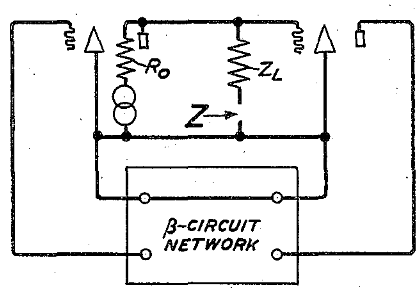


FIG. 28

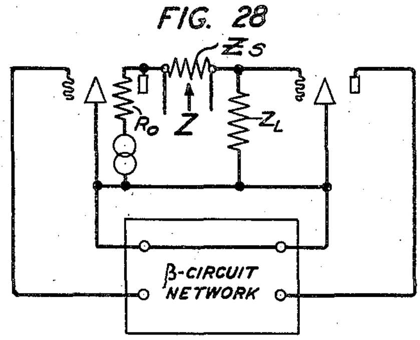
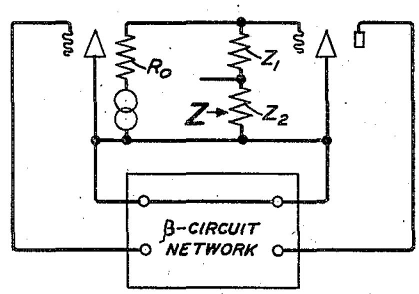


FIG. 29



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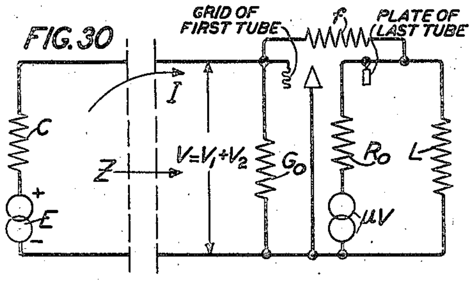
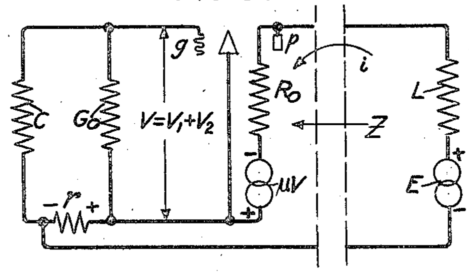
INPUT IMPEDANCE
CONFIGURATION A- VOLTAGE- VOLTAGE

FIG. 30



OUTPUT IMPEDANCE
CONFIGURATION B-CURRENT-CURRENT

FIG. 34



OUTPUT IMPEDANCE
CONFIGURATION A- VOLTAGE- VOLTAGE

FIG. 31

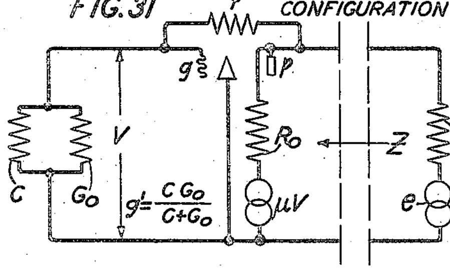
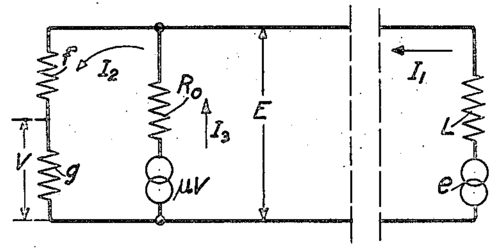


FIG. 31A



FEEDBACK IMPEDANCE
CONFIGURATION A- VOLTAGE-VOLTAGE

FIG. 32

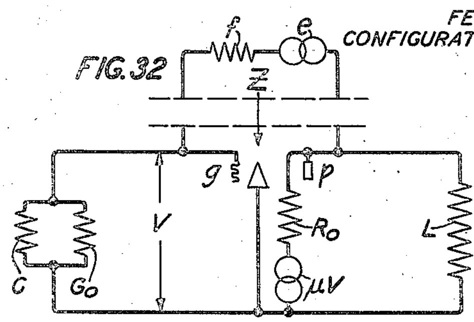
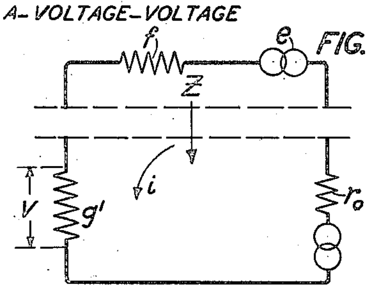


FIG. 32A



INPUT IMPEDANCE
CONFIGURATION B- CURRENT-CURRENT

FIG. 33

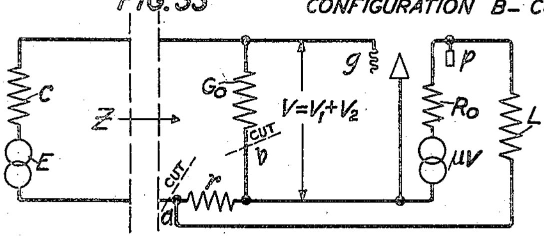
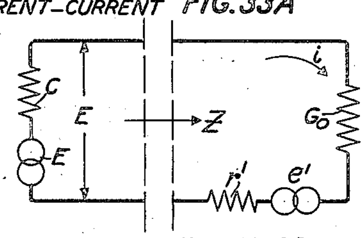


FIG. 33A



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CONFIGURATION A

FIG. 35A

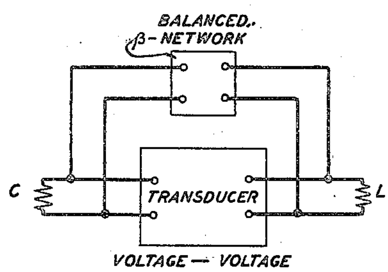
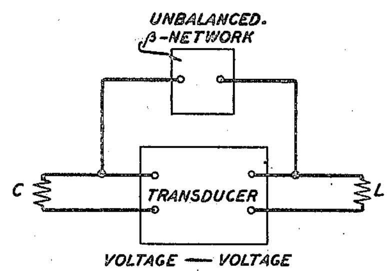


FIG. 35B



CONFIGURATION B

FIG. 35C

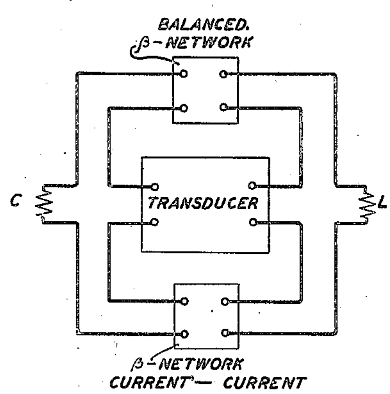


FIG. 35D

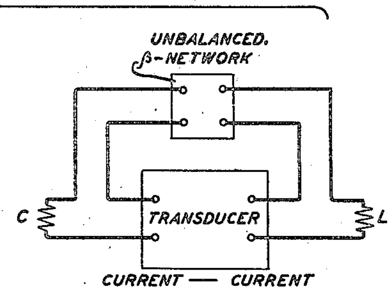
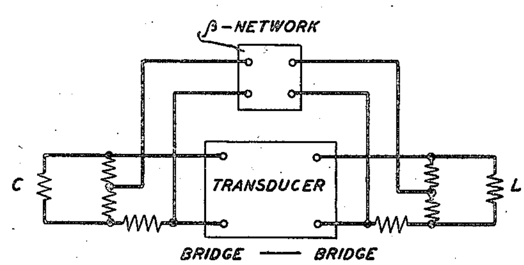


FIG. 35E

CONFIGURATION C



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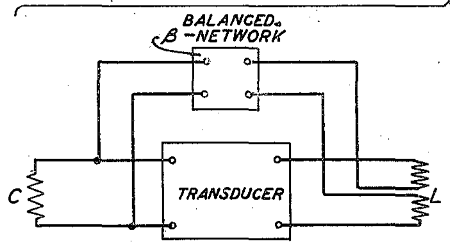
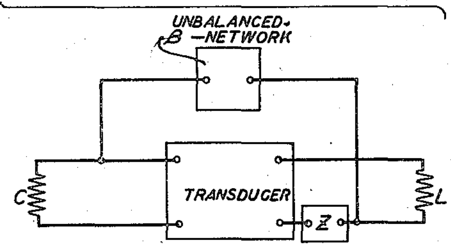
CONFIGURATION D

FIG. 36A



VOLTAGE—CURRENT

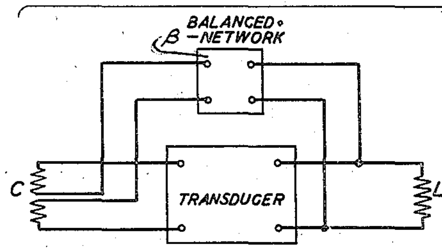
FIG. 36B



VOLTAGE—CURRENT

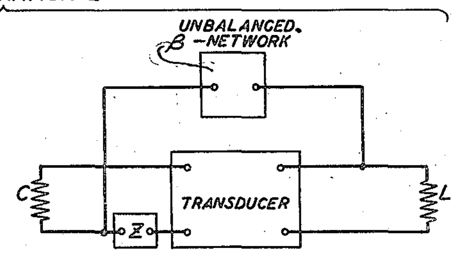
CONFIGURATION E

FIG. 36C



CURRENT—VOLTAGE

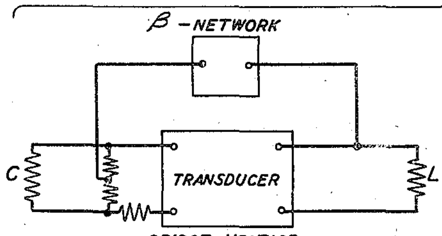
FIG. 36D



CURRENT—VOLTAGE

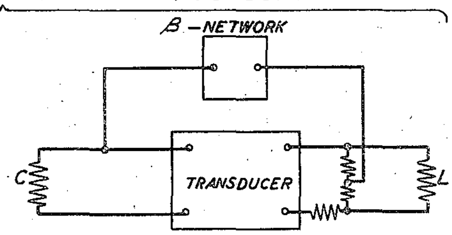
CONFIGURATION F

FIG. 36E



BRIDGE—VOLTAGE

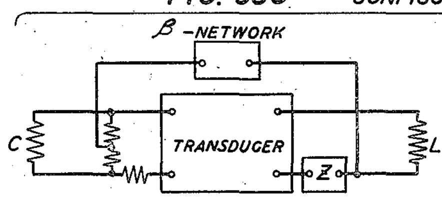
FIG. 36F



VOLTAGE—BRIDGE

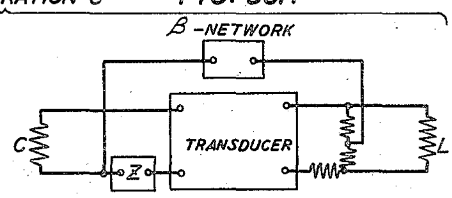
CONFIGURATION G

FIG. 36G



BRIDGE—CURRENT

FIG. 36H



CURRENT—BRIDGE

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FIG. 37

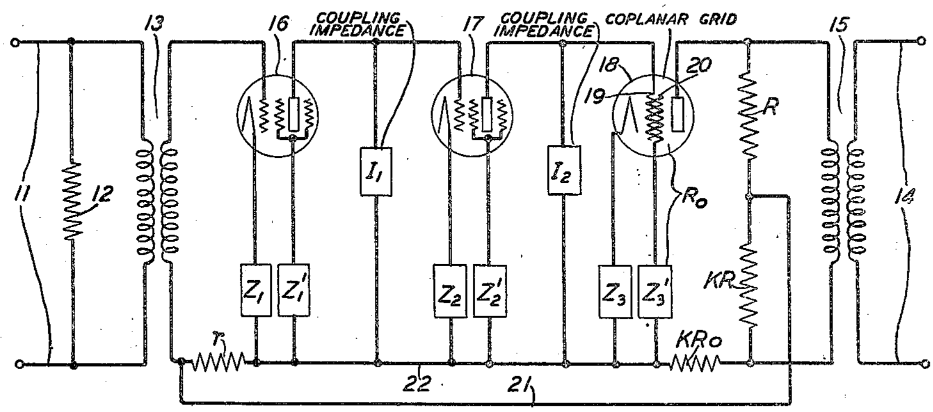

FIG. 38

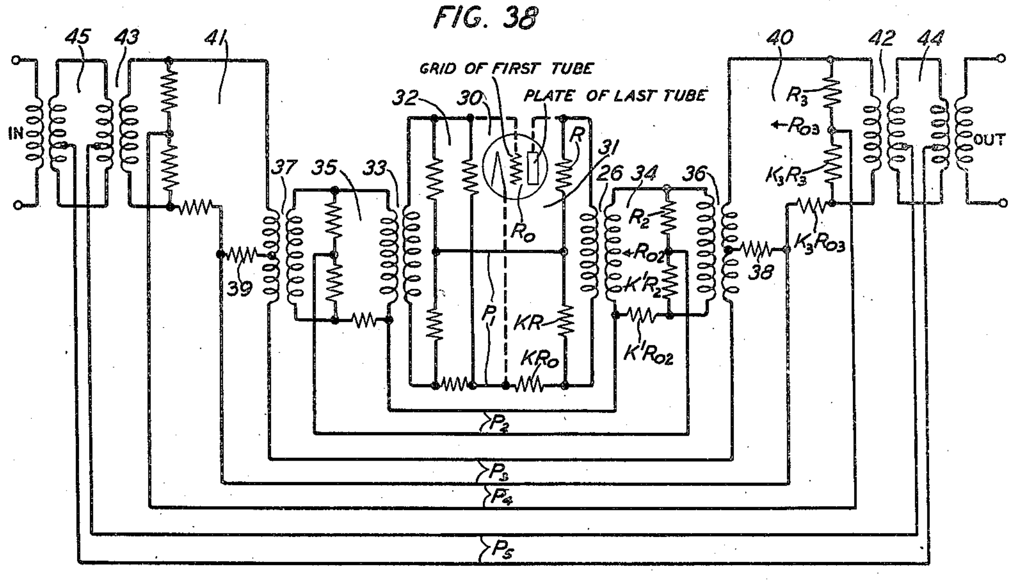

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FIG. 39

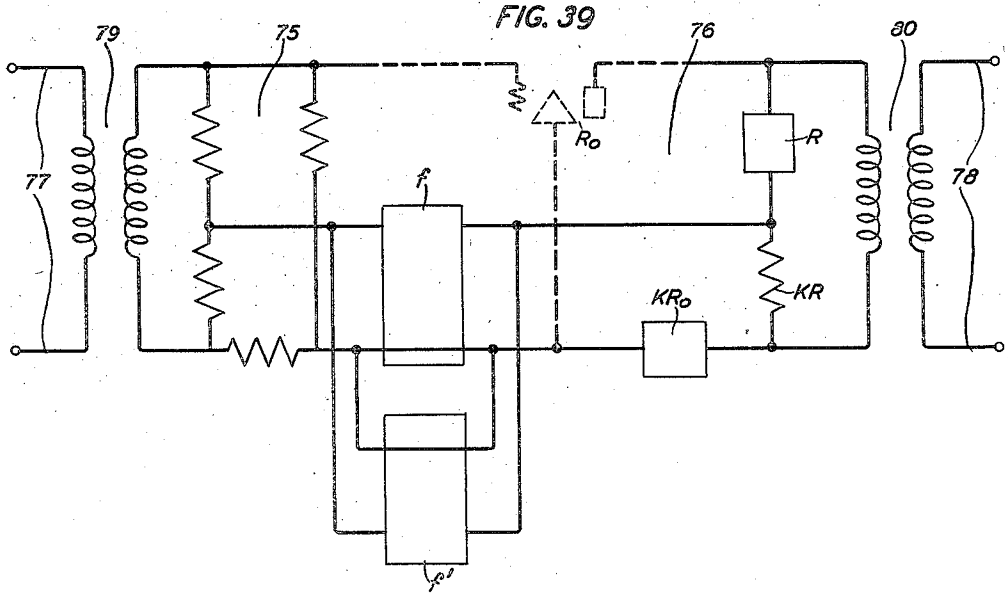

FIG. 40

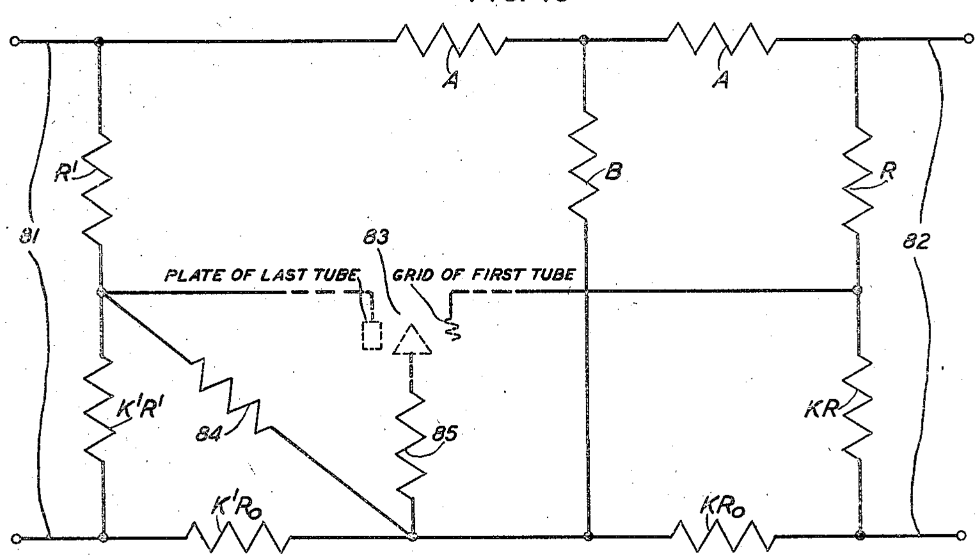

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FIG. 41

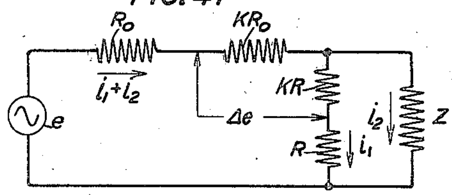


FIG. 42

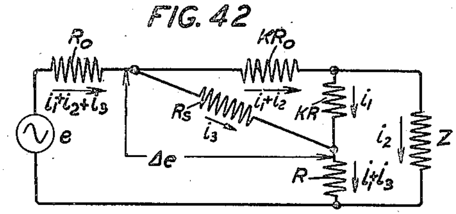


FIG. 43

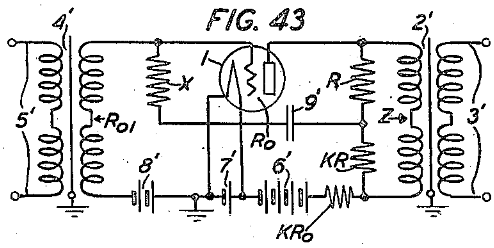


FIG. 43A

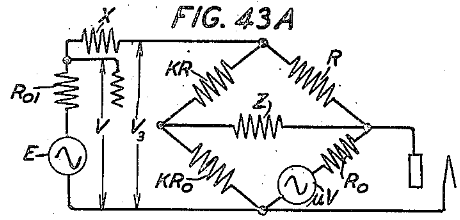


FIG. 43B

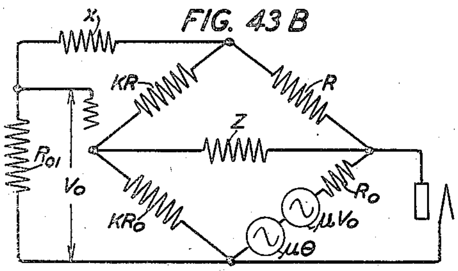


FIG. 43C

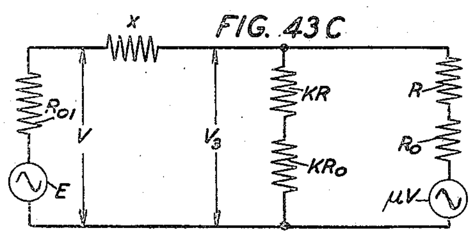


FIG. 43D

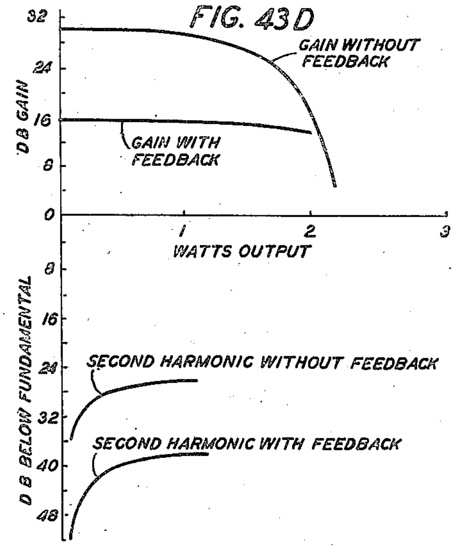
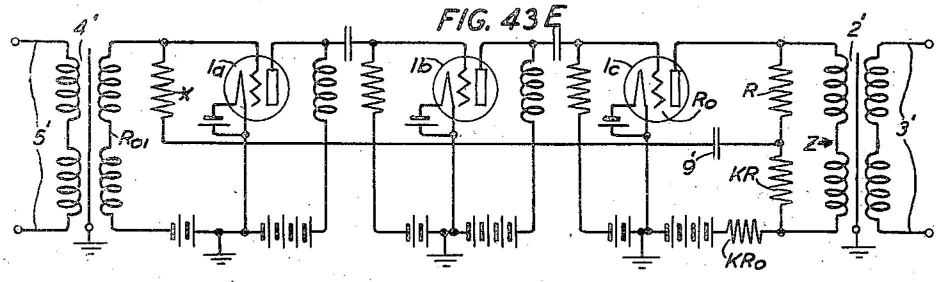


FIG. 43E



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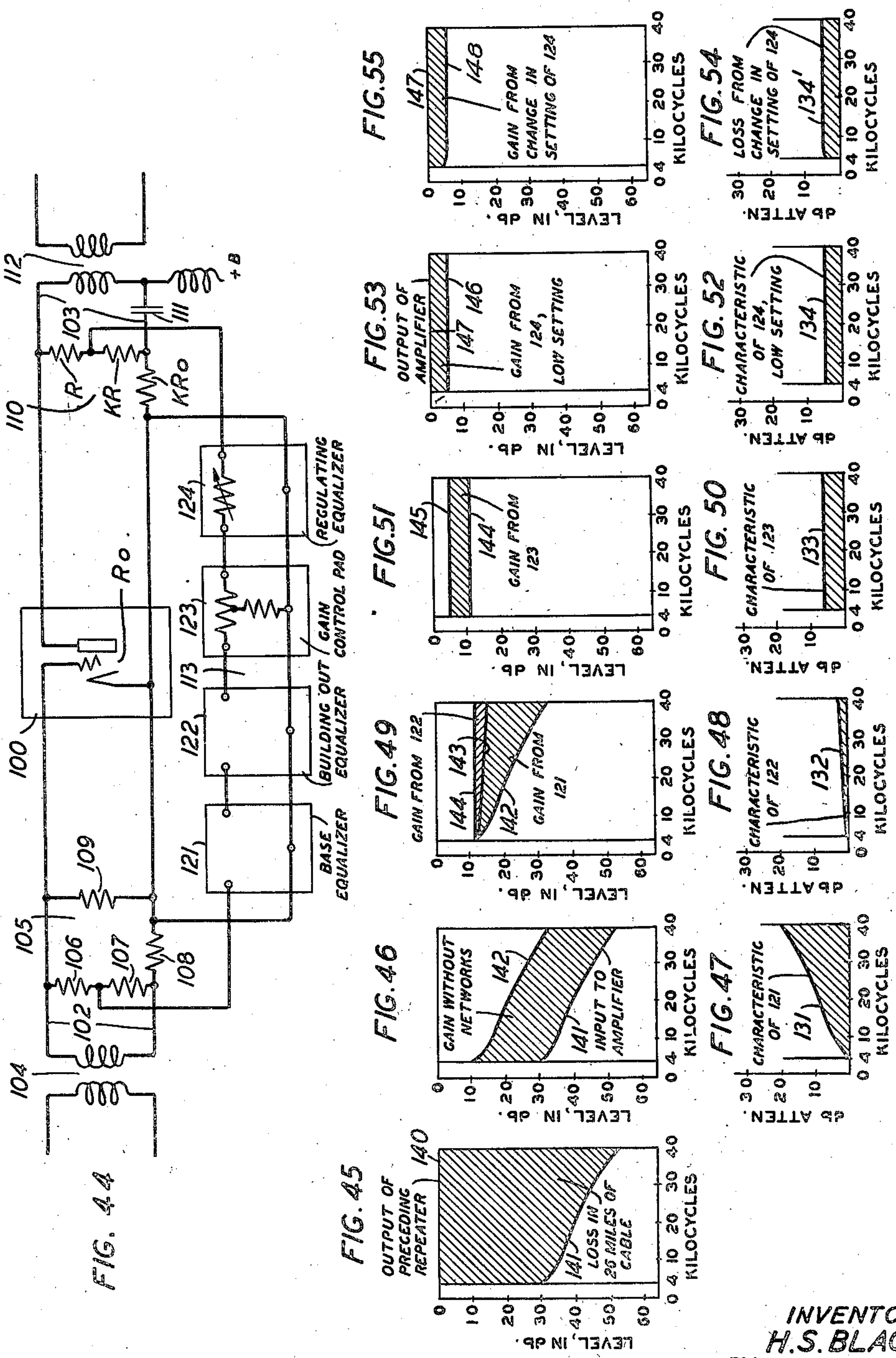

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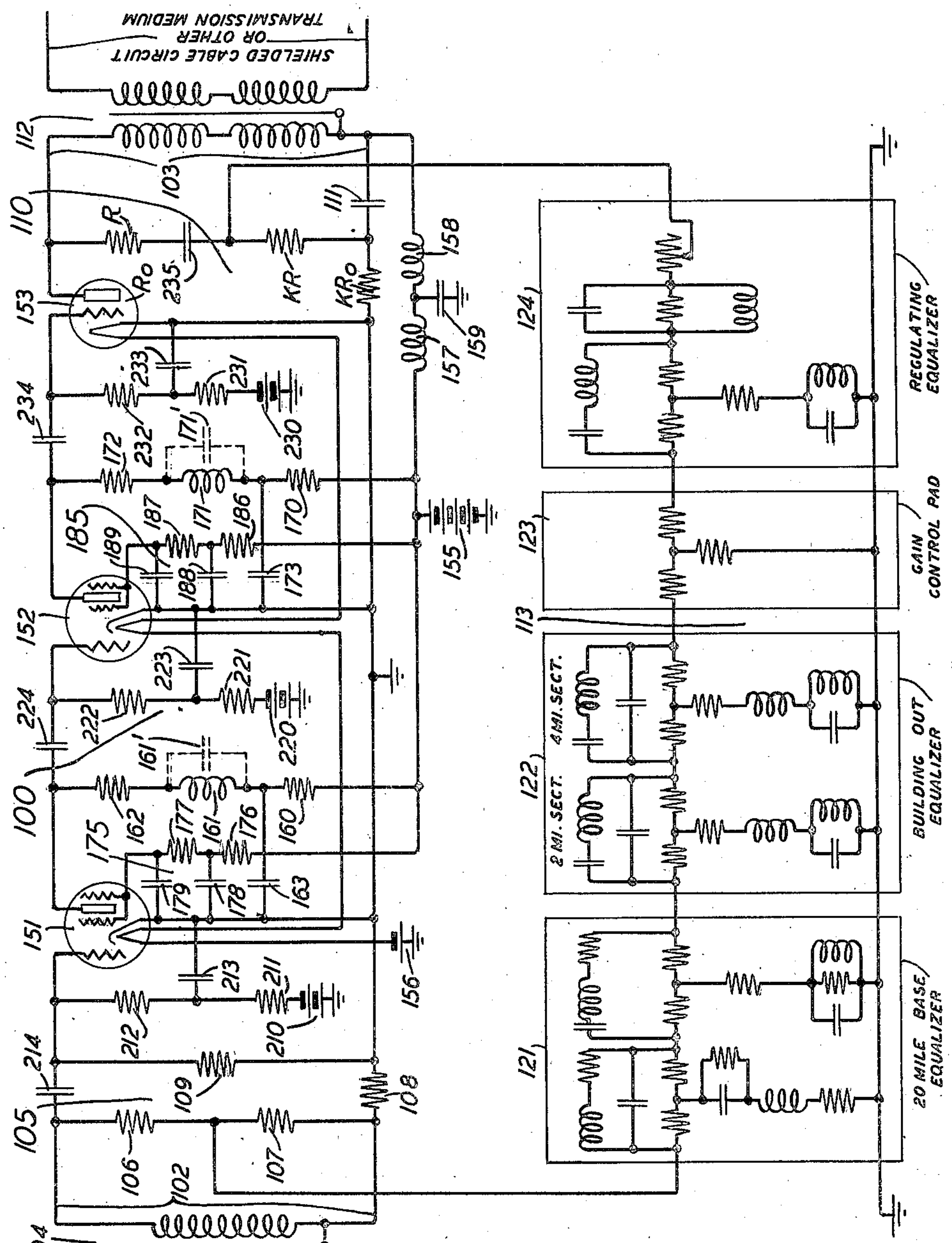

FIG. 56

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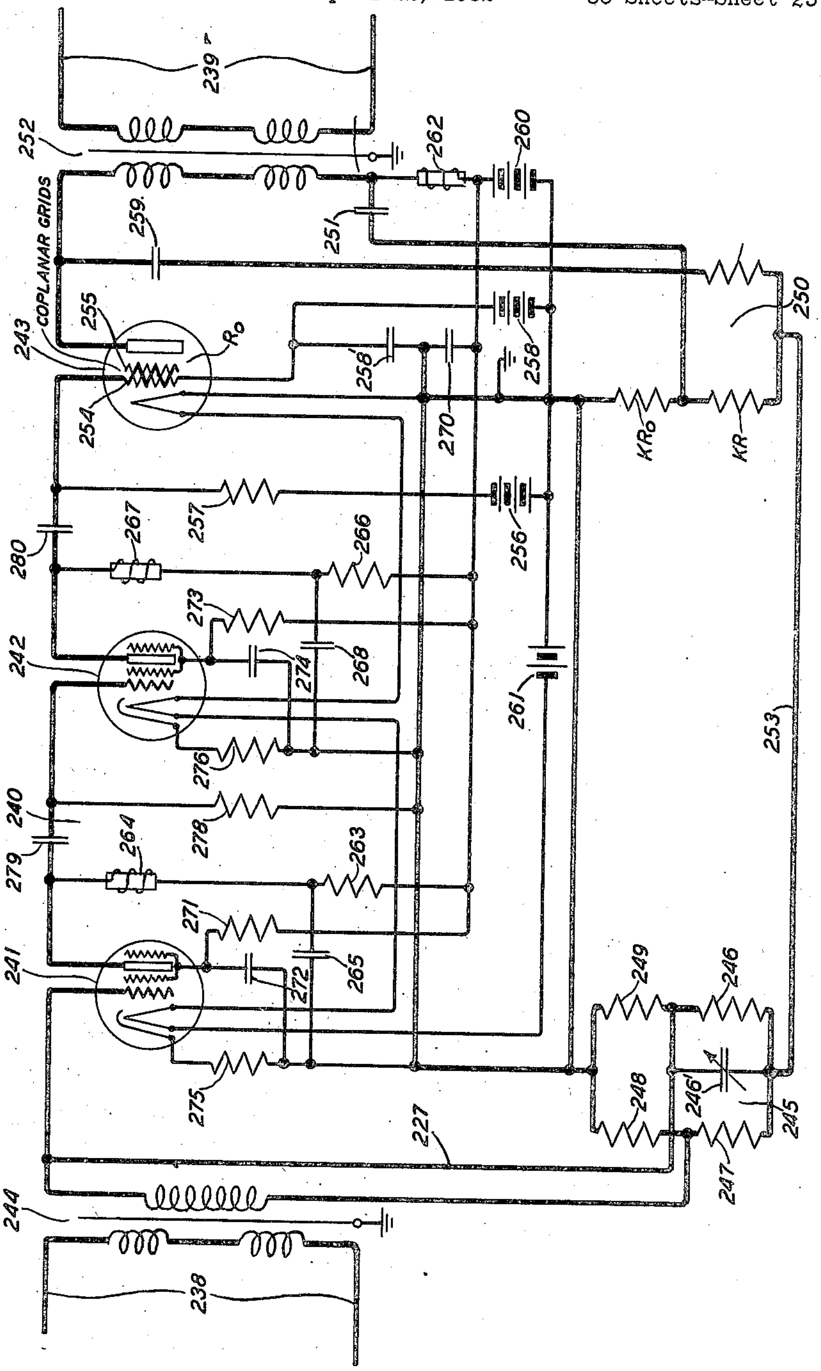

FIG. 57

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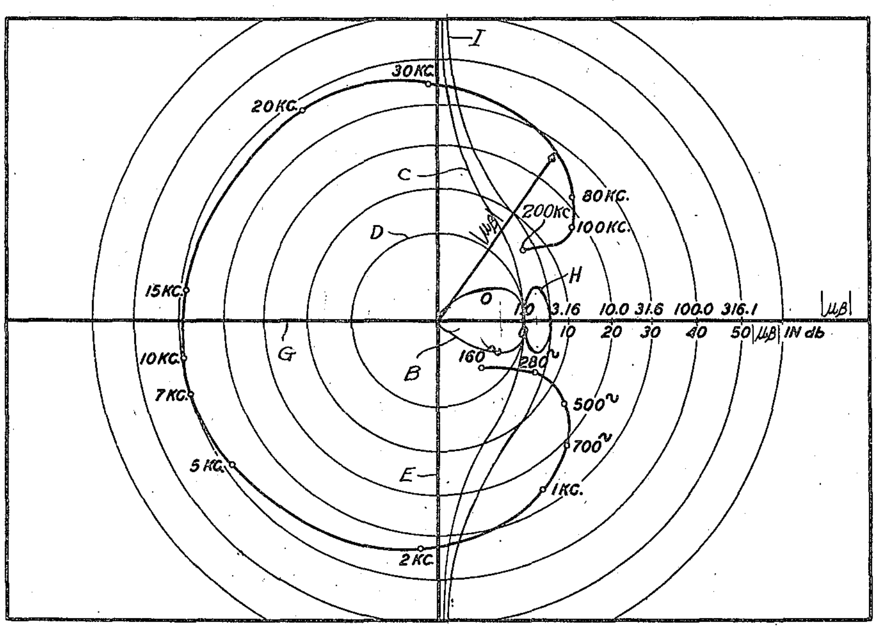
FIG. 58



POLAR PLOT OF $\mu\beta$ FOR THE FEEDBACK AMPLIFIER OF FIG. 57

FIG. 71

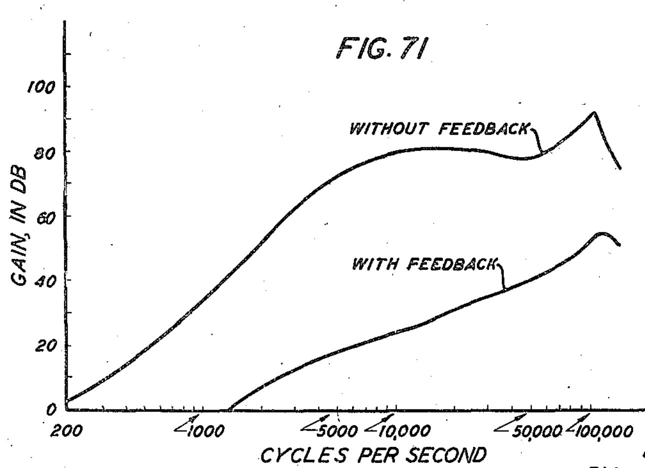


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FIG. 59

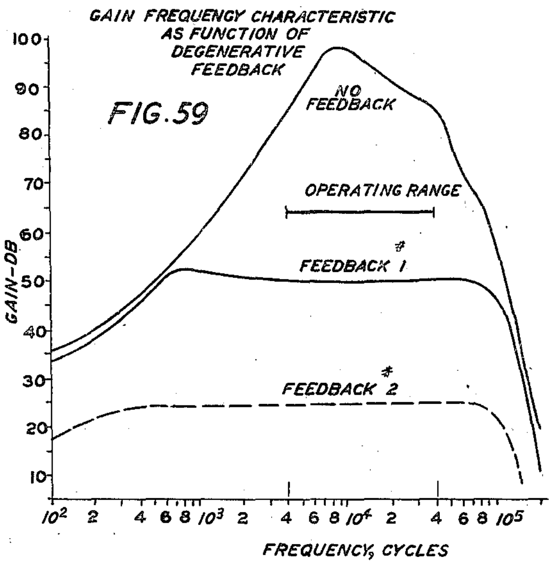

FIG. 60

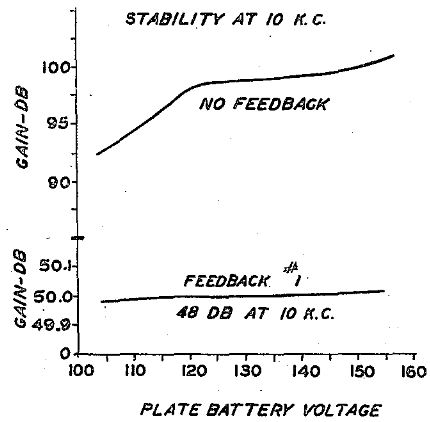

FIG. 61

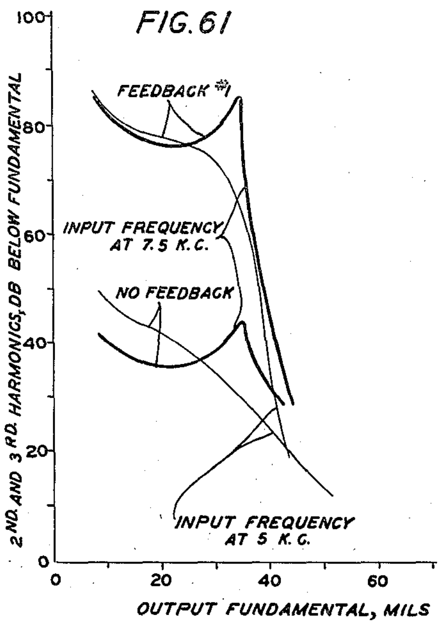

FIG. 62

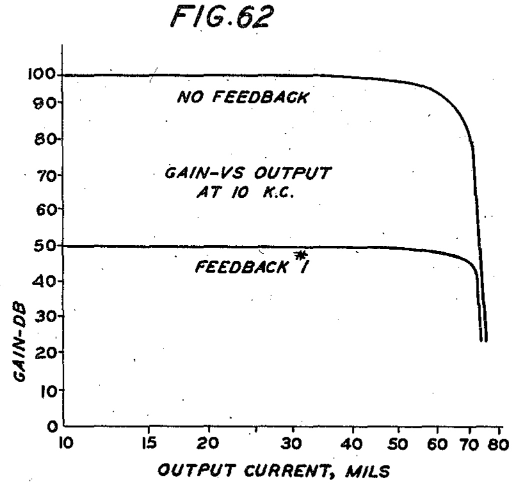

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FIG. 63

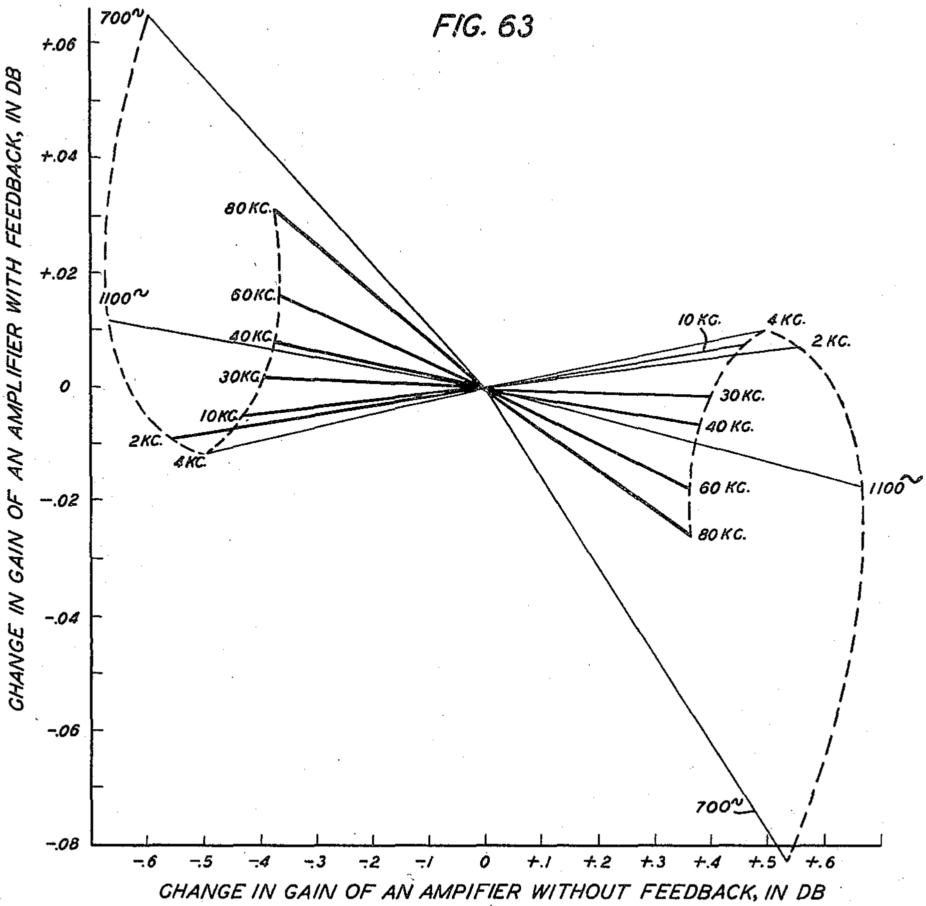
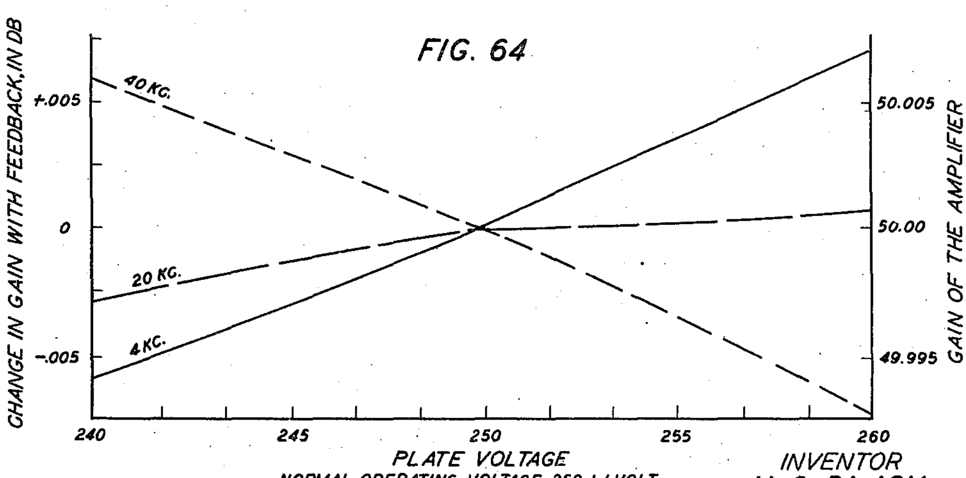


FIG. 64



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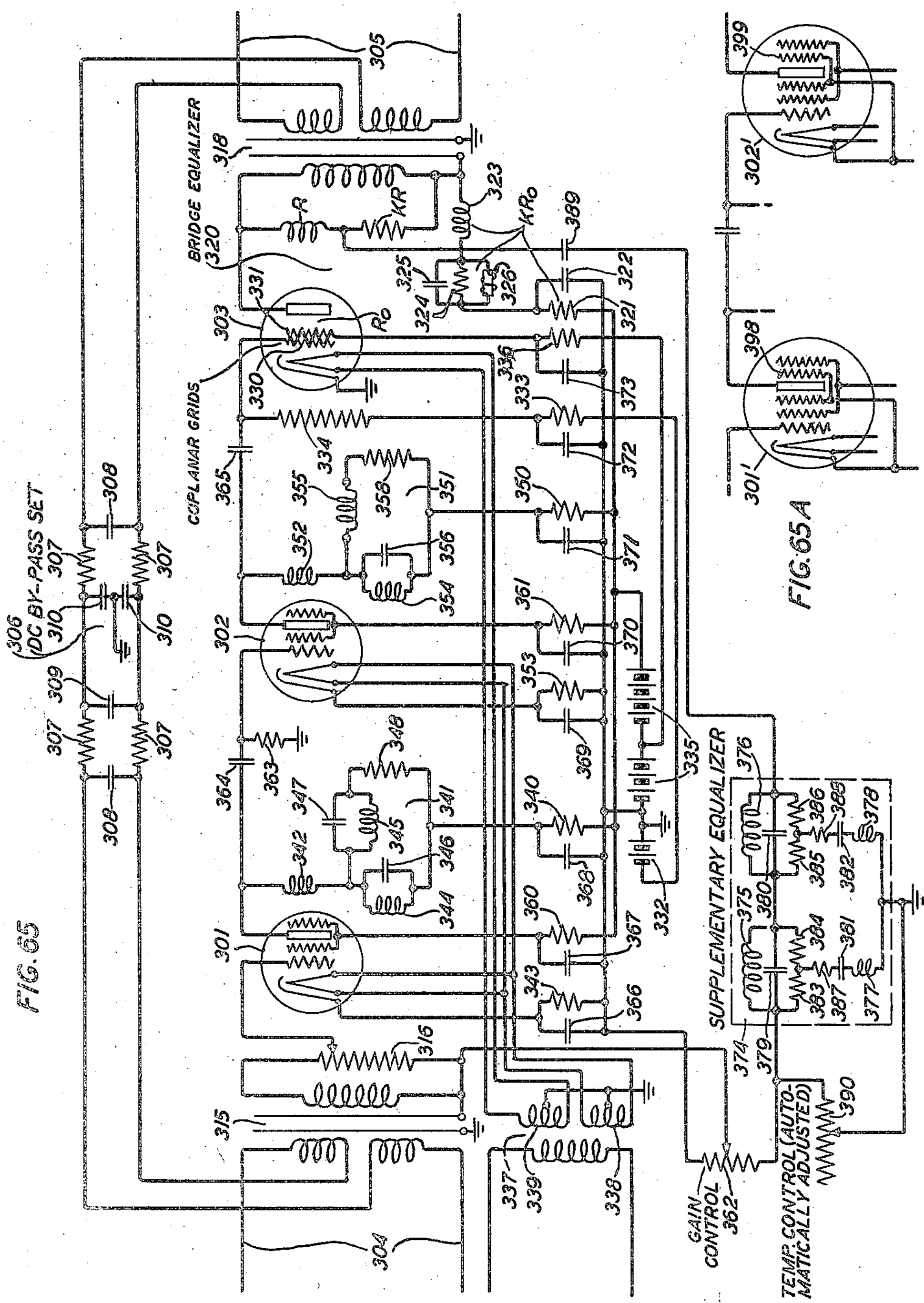

FIG. 65

FIG. 65A

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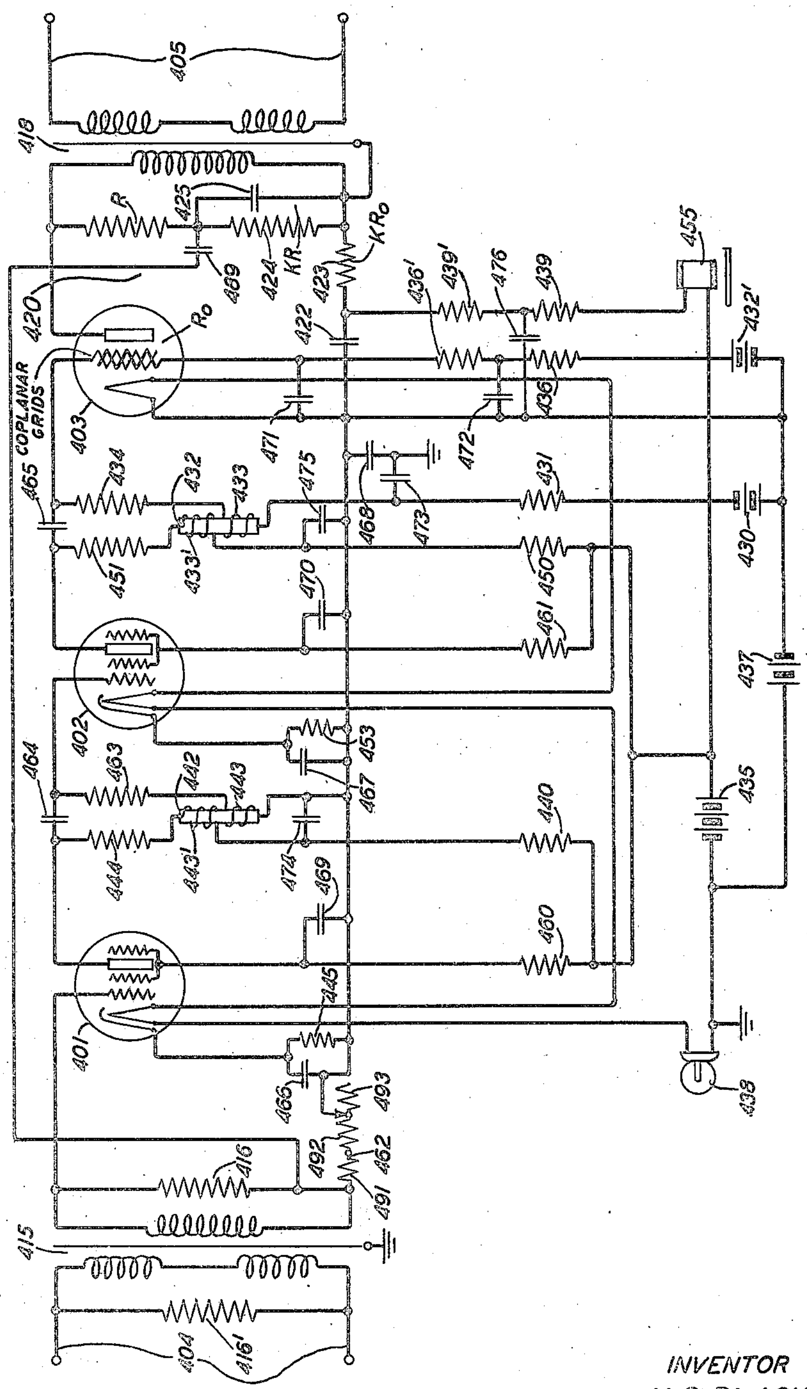

FIG. 66

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FIG. 67

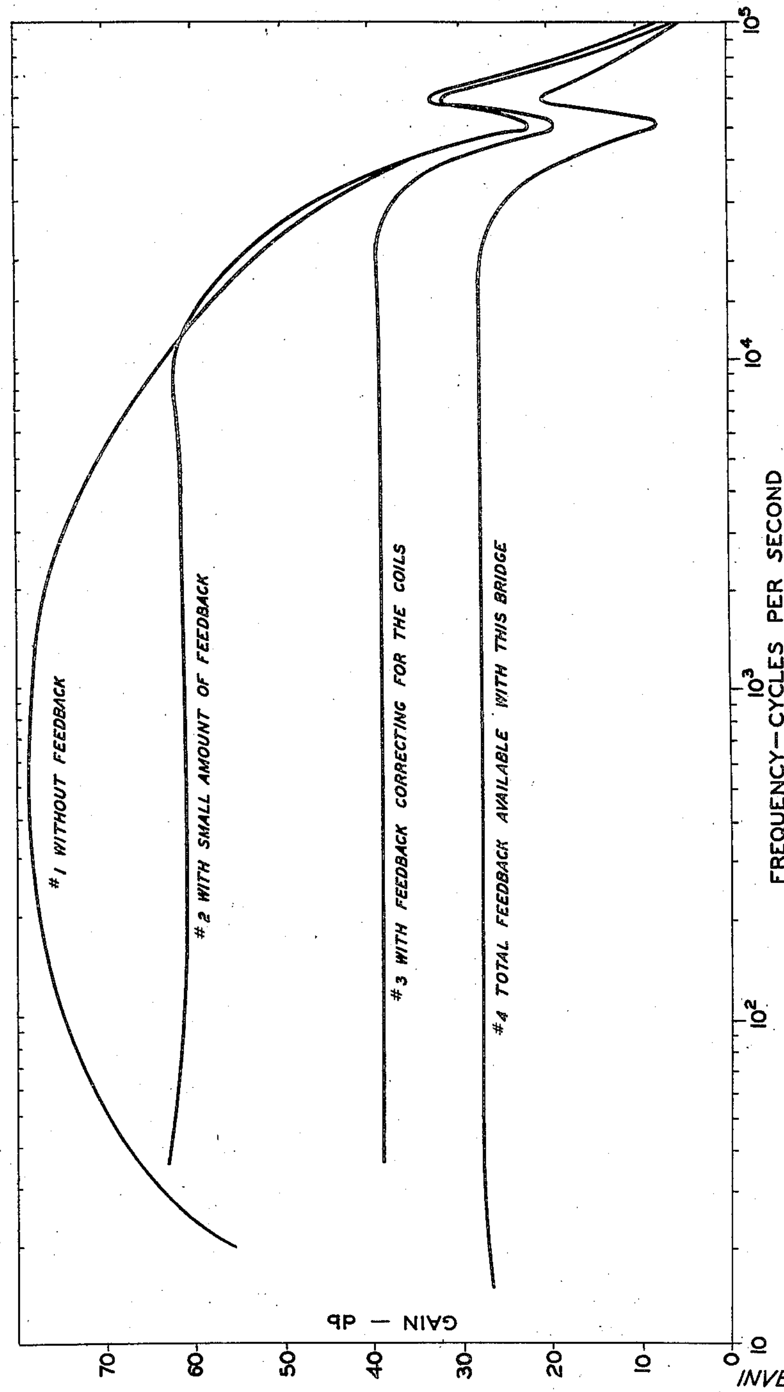

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FIG. 68

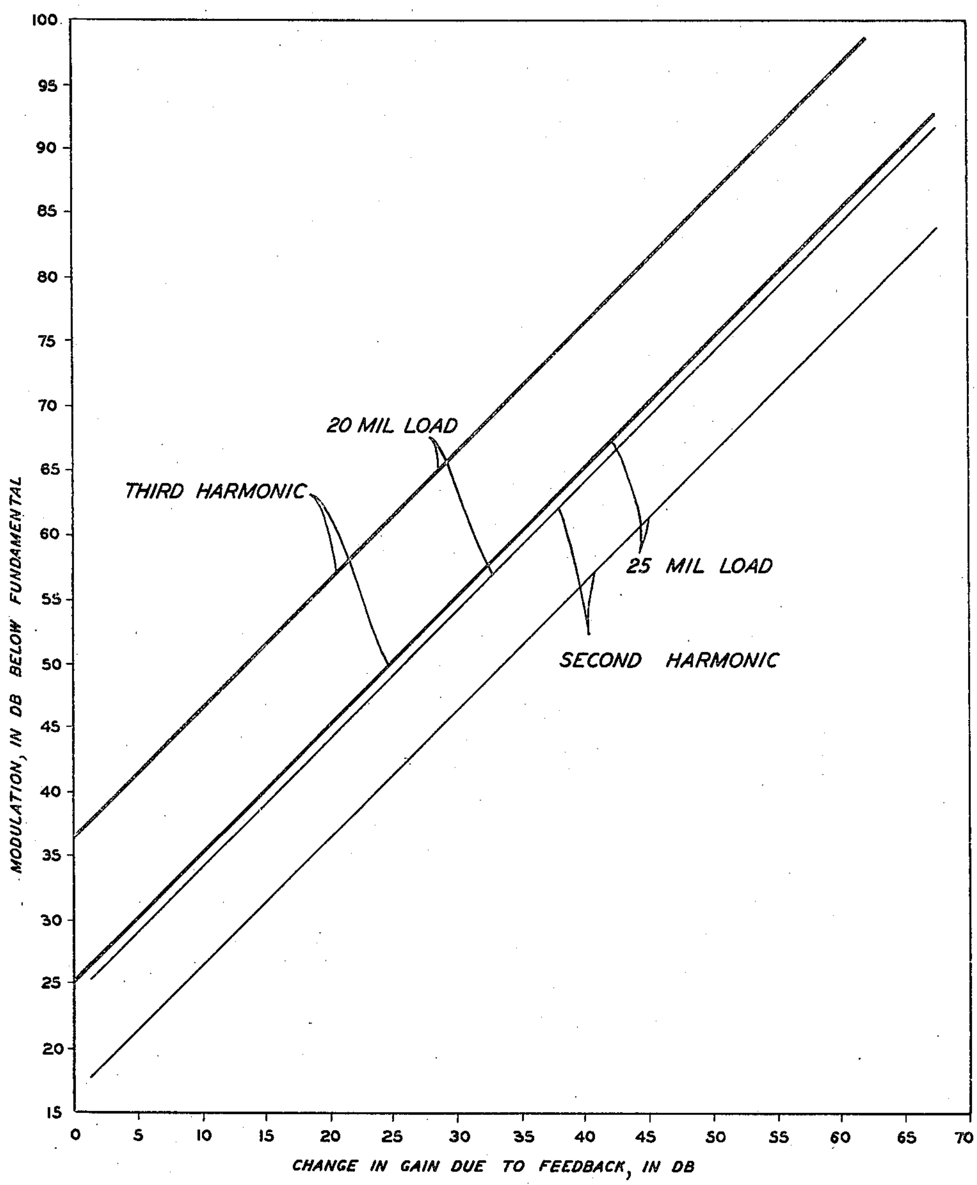

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FIG. 69

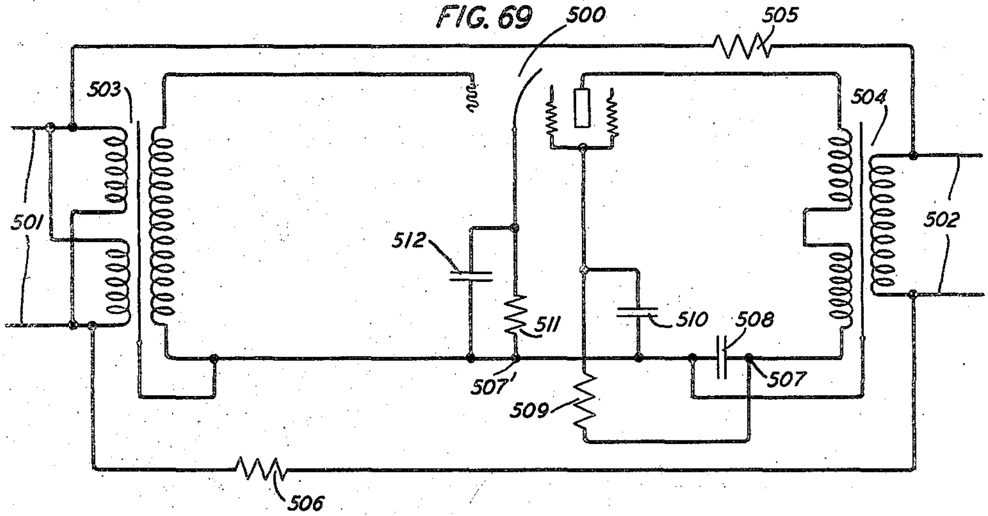

FIG. 70

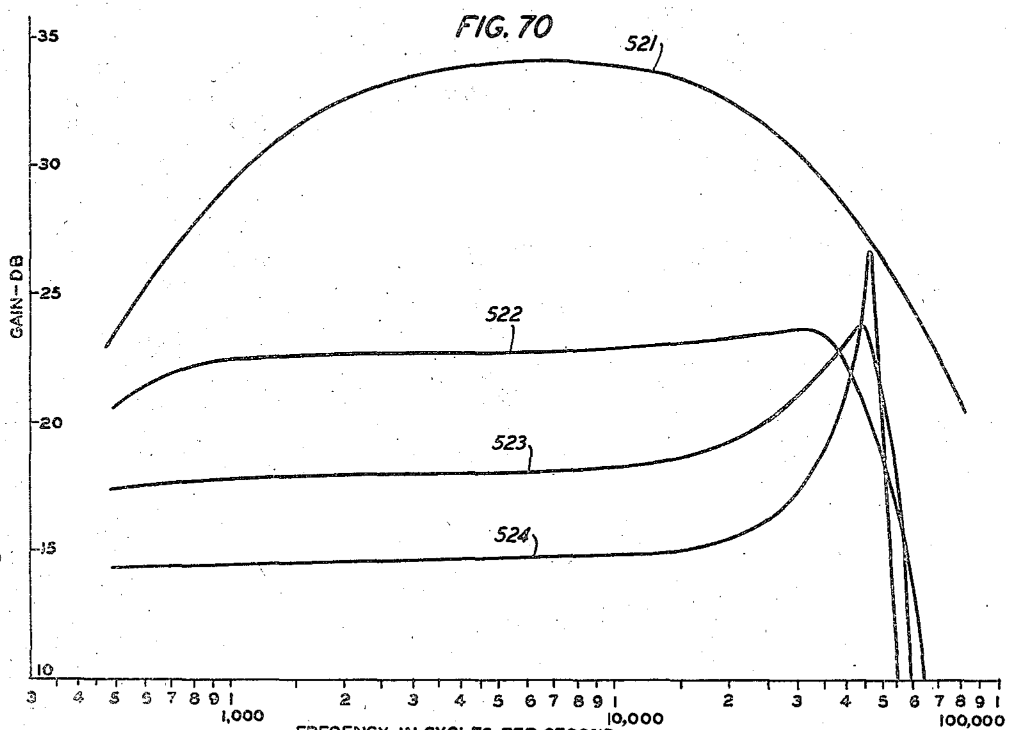

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FIG. 72

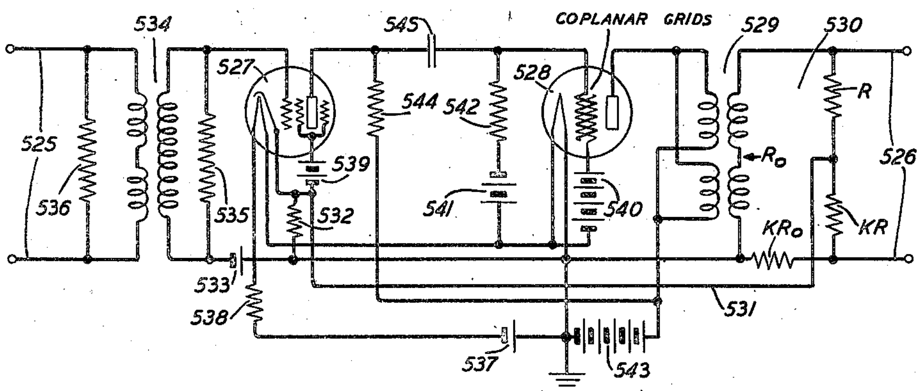

FIG. 73

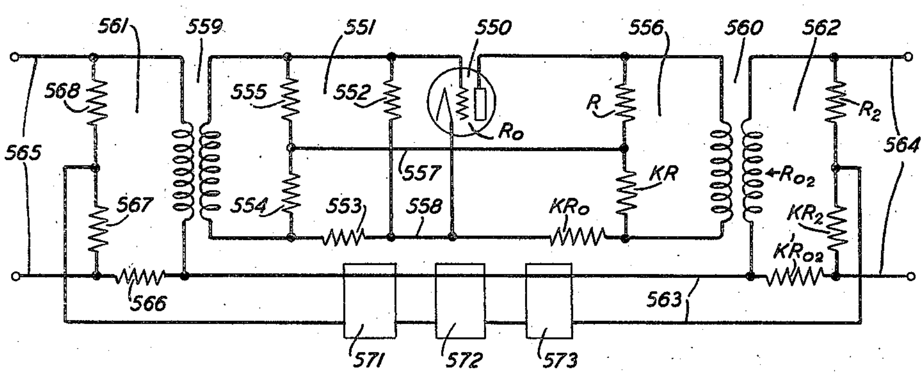

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FIG. 74

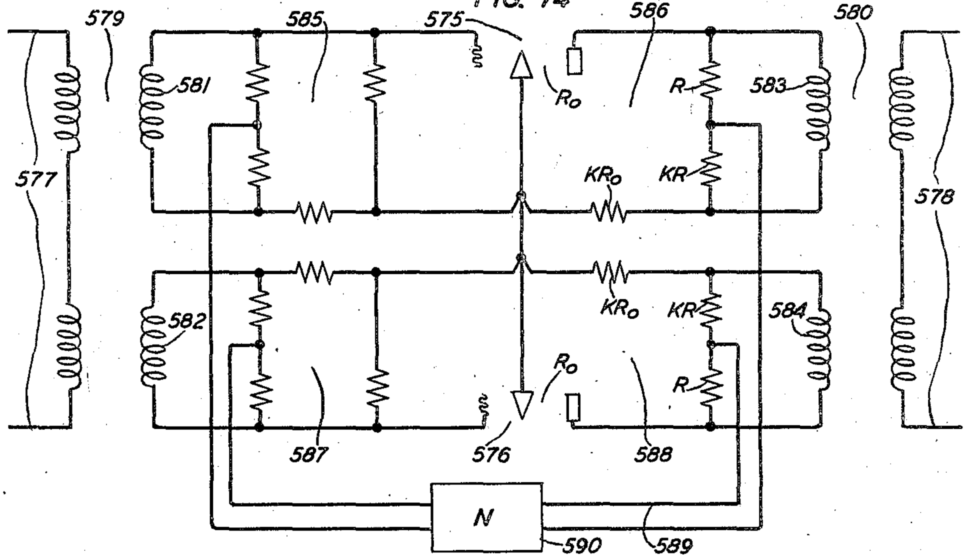


FIG. 75

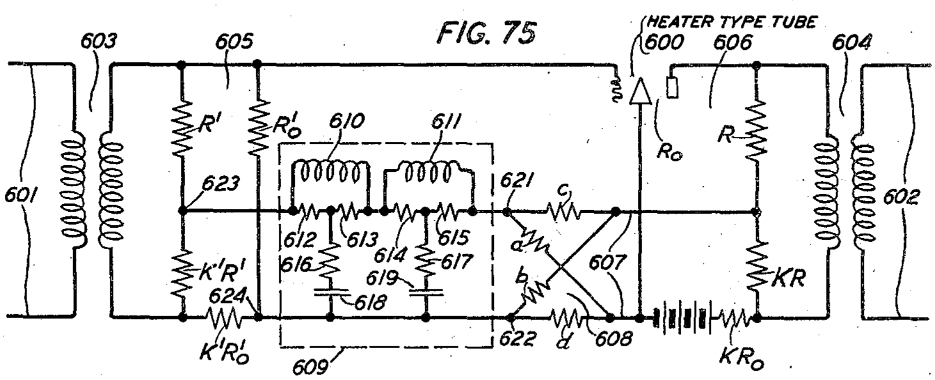
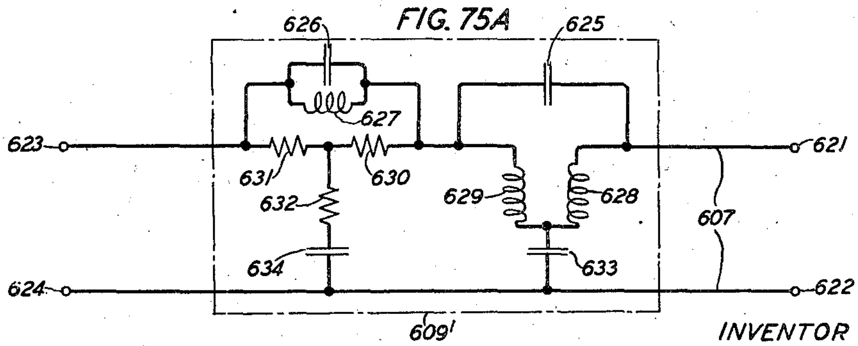


FIG. 75A



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Patented Dec. 21, 1937

2,102,671

UNITED STATES PATENT OFFICE

2,102,671

WAVE TRANSLATION SYSTEM

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Application April 22, 1932, Serial No. 606,871
In Canada May 21, 1929

126 Claims. (Cl. 178—44)

This application is in part a continuation of application Serial No. 298,155, filed August 8, 1928, for Wave translation systems and in part a continuation of application Serial No. 439,205, filed March 26, 1930, for Wave translation systems.

The present invention relates to wave transmission apparatus or systems, and more particularly to such apparatus or systems employing an amplifying element provided with a feedback path. The principal field of application of the invention is in apparatus or systems employing negative feedback.

For purposes of simplified introductory discussion and leaving exact definition for a later point, the elements of a system with feedback are: an amplifying element having an input and an output; and a coupling or path for returning some of the output wave to the input of the amplifying element.

The invention is applicable to any kind of wave transmission such as electrical, mechanical or acoustical, and thus far in the description the terms used have been generic to all such systems. The invention will be disclosed herein, however, as specifically applied to electrical systems, it being understood that the principles involved are equally applicable to other types of wave transmission and that the generic claims are intended to include electrical and other than electrical wave systems and apparatus.

Reverting now to the elemental feedback system above described, and taking a typical electrical case, for illustration, the amplifying element may be a grid-controlled discharge tube of ordinary type. The input circuit leads up to the input side from a suitable source of input waves to be amplified, for example, signal waves. The feedback is connected from the output circuit to the input circuit. It will be noted that the waves actually applied to the grid come from two circuit branches, identified as the incoming circuit (from the signal or other wave source) and the feedback circuit. Of course, each of these two waves, the incoming or signal wave and the fed-back wave, could be thought of as being separately applied to the grid, but it will be simpler to think of a summation wave as the voltage actually effective on the grid. Thus, there are three waves to be considered in relation to the input side of the amplifier, (1) the incoming or original wave to be amplified, (2) the fed-back wave, and (3) the voltage wave effective on the grid, this latter being the resultant produced by the algebraic addition of the other two.

The prior art has recognized as generic types, positive feedback and negative feedback. Positive feedbacks have been classified into the so-called regenerative amplifier and the oscillation generator which, from a practical standpoint, are sharply distinguished from each other.

If we picture the fed-back wave and the incoming wave as separately adjustable, we may imagine an increasing amount of fed-back waves in a direction to augment the effect of the incoming wave with a corresponding decrease of the incoming wave. The adjustments may continue until the input wave has been entirely replaced by the fed-back wave. That is, the incoming wave has been reduced to zero and the incoming circuit can be disconnected from the amplifier. The waves in the circuit then become self-sustained and we have the familiar case of the wave generator developing and maintaining continuous oscillations, in a closed loop consisting of the input circuit, the amplifier, the output circuit and the feedback circuit back to the input circuit.

If a smaller amount of positive feedback is employed than is required to produce sustained oscillations, we have the case of the regenerative amplifier. Some of the incoming signal wave after amplification is again put back on the input circuit in such manner as to augment the original incoming signal so that reamplification occurs in the same tube, causing the signal to build up to relatively higher value than could be done by traversing the amplifying element but once. The useful limit of amplification by such a circuit is reached when a further increase in the degree of feedback would result in producing self-oscillation or when the circuit is sufficiently close to the oscillating condition to cause an intolerable amount of signal distortion.

A broad practical distinction between the oscillator and the regenerative amplifier is that in the latter the output current remains under the control of the incoming signal and must follow its variations of starting, stopping, growing stronger or weaker and exhibiting the characteristic quality of the signal, whereas in the oscillator the wave circulating around the regenerative loop is self-determined and is beyond the control of any signal or other input wave.

Turning, now, from positive feedback to negative feedback we find that the principal field of use of negative feedback has been in radio frequency amplifiers.

A radio frequency amplified is a familiar instance of a circuit in which there may exist an inherent tendency toward self-oscillation because of a positive feedback produced by inductive or capacitive coupling between elements of the output and input circuits even where great care in design is exercised to reduce such coupling. These effects become more pronounced as the frequencies become higher and higher. The tendency toward self-oscillation in such circuits places a limit on the amount of amplification that can be used.

Negative feedback has commonly been applied in the prior art in radio frequency amplifiers to oppose the inherent positive feedback. The negative feedback in all such cases has had as its purpose the reduction of the positive feedback. If we imagine that the negative feedback is increased from an infinitesimal amount in any given case, it reaches its optimum value in opposing positive feedback when it just equals the positive feedback. At that point, the net or resultant total feedback is zero and the effect is that of rendering the amplifier a strictly unilateral circuit or one having no feedback, either positive or negative.

In contrast to the negative feedback of the prior art in which, as discussed above, the resultant total feedback is substantially zero, the invention uses negative feedback for an entirely different purpose and in very much larger amounts to achieve improved results in wave translation, e. g., amplification, not envisaged in the prior art.

Applicant has discovered how to use larger amounts of negative feedback than were contemplated by prior art workers with a new and important kind of improvement in tube operation. One improvement is in lowered distortion arising in the amplifier. Another improvement is greater constancy of operation, in particular a more nearly constant gain despite variable factors such as ordinarily would influence the gain. Various other operating characteristics of the circuit are likewise rendered more nearly constant. Applicant has discovered that these improvements are attained in proportion to the sacrifice that is made in amplifier gain, and that by constructing a circuit with excess gain and reducing the gain by negative feedback, any desired degree of linearity between output and input and any desired degree of constancy or stability of operating characteristics can be realized, the limiting factor being in the amount of gain that can be attained rather than any limitation in the method of improvement provided by the invention.

It should be obvious from what has been said that the essential elements of the invention can be embodied in a circuit of quite simple configuration. While, in some elementary forms, the diagram of a circuit incorporating the invention may appear similar to prior art circuit diagrams, this resemblance is superficial since the mode of operation and character of results attained are markedly different from the prior art. The proportioning of the circuit elements must be such as to permit the necessary amounts of amplification and feedback ratios with proper phase relations.

Also, from what has been said, it is apparent that applicant uses negative feedback for a purpose quite different from that of the prior art which was to prevent self-oscillation or "singing". To make this clearer, applicant's invention is not concerned, except in a very secondary way (to be explained later on) with the singing tendency of a circuit. Its primary purpose has no relation to the phenomenon of self-oscillation. If amplifiers could be built exactly like present day amplifiers in all respects except that they were absolutely free of any tendency of self-oscillation regardless of how high their gains might be pushed, it is generally true that there would still be as great need for the present invention. The "perfect" amplifier is one in which the output wave is in all respects an exact replica of the input wave multiplied by some linear factor. The singing tendency is only one of several difficulties encountered in an amplifier. Actually, amplifiers produce distortion components along with the output fundamental components. These modulation or distortion components are mingled with the fundamental wave and detract from its purity. In multiplex carrier systems the distorting effect gives rise to cross-talk and, in any high quality system, it constitutes a limiting factor of design.

Applicant has discovered that distortion can be reduced in any amplifying system by use of negative feedback in accordance with the invention.

Another difficulty in amplifier operation is instability, not used here as meaning the singing tendency but rather signifying constancy of operation as an amplifier with changes in battery voltages, temperature, apparatus changes including changes in tubes, aging and kindred causes. Such instability is a limiting factor where, for example, a large number of repeaters are operated in tandem in a line. A simultaneous tendency toward increased gain by several or all of the repeaters might be disastrous. Without a means of preventing such changes, they become a limiting factor of design.

Applicant has discovered that the stability of operation of an amplifier can be greatly improved by the use of negative feedback.

These two kinds of improvement realizable by use of negative feedback are given by way of example. Other advantages will appear from the fuller description to follow.

The general object of the invention is to employ feedback to improve the operation of a circuit as by decreasing distortion or increasing stability.

The invention, in one aspect, comprises use of enough negative feedback to increase the linearity of an amplifier.

The invention, in one aspect, comprises use of enough negative feedback to improve stability of an amplifier against changes in operating characteristic with changes in constants of component parts of the amplifier circuit or apparatus.

Reverting again to the elemental feedback circuit discussed above, it was pointed out that the prior art used negative feedback to cancel positive feedback giving a resulting circuit with zero feedback. In such cases, the total fed-back wave was made up of some positively fed-back wave and enough negatively fed-back wave to bring the total fed-back wave to zero. Thus, the voltage wave effective on the grid was exactly equal to the incoming wave, nothing being either added to or subtracted from the incoming wave in such case. It was realized that with an increased amount of negative feedback, the voltage wave effective on the grid could be made slightly less than the incoming wave, with consequently lowered gain. Except as this small net negative feedback afforded a margin of safety against self-oscillation it was considered a detriment since it lowered the gain, and there was no realization that, apart from opposing self-oscillation, there was any advantage from the standpoint of amplifier operation in reducing the gain.

Moreover, while various circuits were known involving connections between input and output for suppressing or eliminating some undesired wave component in the system there was no prior realization, as far as applicant is aware, that negative feedback could be used to reduce modulation, improve linearity and increase stability of the circuit for the transmitted waves. Some writers of recognized standing in the field of positive feedback circuits even went so far as to point out that the customary analysis of such circuits led to the conclusion that no greater degree of negative feedback was possible than a degree exactly equal to the positive feedback which corresponded to the singing condition. Such a view precluded any expectation that large amounts of negative feedback could be used in an amplifier for any purpose. According to that view, a limit to the amount of negative feedback would be reached at the point where the voltage effective on the grid is just equal and opposite to the fed-back voltage, for any increase in the fed-back voltage beyond that point would upset the stability of the circuit as an amplifier and make it a generator of oscillations.

Contrary to the teachings of the prior art, applicant utilizes such a large value of negative feedback that it not only may equal but greatly exceed the amplitude of the wave that is effective on the grid. That is, the voltage wave that is effective on the grid is not only smaller than the incoming voltage wave but is smaller than the fed-back voltage wave. In effect, this means that a small wave effective on the grid is controlling a cycle of operations involving waves of much greater magnitude, namely, a larger incoming wave and a larger fed-back wave. Far from resulting in a liability of self-oscillation, applicant has discovered that the greater the negative feedback ratio, the more exact is the correspondence in all respects between the output wave and the incoming wave, so that it may be said that the more complete is the control of the output wave by the incoming wave. The result is greater linearity of amplification and greater constancy of operating characteristics as regards the influence of circuit variables.

In an attempt to gain some physical concept of the action which takes place in the circuit resulting in these improvements, the following elementary picture may be helpful.

It was stated above that the amplified signal wave in the output of an amplifier is accompanied by distortion produced in the tube. An assumption, justified by experience, will now be made that the ratio which this distortion bears to the signal is a function of the amplitude of the output signal, other things being equal. That is, with a given tube and circuit, as the signal output is increased, the percentage of distortion increases. It is further to be noted that a distortion component appearing in the output of an amplifier can be reduced by application to the input circuit of some of the distortion component in reversed phase.

Referring to the simple system with feedback already considered, let it be imagined that a simultaneous control can be made in the amplitudes of the incoming signal and of the fed-back wave such as always to keep the wave effective on the grid at a constant amplitude. Starting out with no fed-back wave, the voltage effective on the grid is that of the incoming signal unmodified by feedback. There is, then, a given amount of output signal and distortion. If, now, negative feedback is gradually introduced in increasing amount and at the same time the incoming signal is increased by an exactly corresponding amount, it is clear that the voltage effective on the grid due to the signal alone remains unchanged, and that, therefore, the signal output remains unchanged in amplitude. By virtue of the negative feedback, however, some of the distortion is being fed back to the grid in such sense as to reduce the distortion appearing in the output. The result is less distortion with no diminution in signal output, a net improvement in linearity of the circuit.

The apparatus for increasing the negatively fed-back wave (e. g., from an infinitesimal or small value) might be visualized as an amplifier of variable gain in the feedback path. Likewise, the apparatus for producing corresponding increase in the signal in the incoming circuit may be thought of as an amplifier of variable gain. Since the coordinated changes assumed to take place in these two amplifiers is a simultaneous increase in their gains by an exactly corresponding amount, the next step in developing the picture is to visualize these two as one and the same amplifier through which both the incoming signal and the fed-back wave are transmitted. This amplifier may be pictured as introduced just ahead of the existing amplifier, that is, between the input to that amplifier and the junction between the incoming circuit and the feedback circuit.

Summarizing the foregoing, it is recalled that an amplifier was first assumed in which there was at the start no feedback, and in which there was a given output of signal and distortion. It was shown that the distortion was reduced relative to the signal by a circuit which is found to differ from the circuit first assumed by the addition of a negative feedback and by an increase in the total gain of the amplifier, still keeping the amplitude of the signal effective on the grid the same as before and consequently the signal output the same. In other words, the gain in the amplifying path was increased but the increase was nullified by a negative feedback.

The foregoing may serve as illustrating in a non-technical and approximate way what is meant by securing greater linearity in an amplifier by providing it with excess gain over that eventually needed and using a negative feedback to cancel the excess gain.

The degree of stability and linearity obtainable is controlled by the feedback, and the feedback can, for example, improve the stability a thousandfold and reduce harmonic currents in the output to one-thousandth and their energy to one-millionth of the values without feedback. (These values are merely illustrative, and not limiting.) Thus the feedback can be highly useful for example, in causing the magnified output to be a faithful reproduction of a signal input. Linearity is essential to such fidelity of reproduction and requires constancy of amplification independent of input amplitude. This implies freedom from harmonic production and other unwanted modulation effects. Stability of transmission is an additional desirable attribute and in certain applications necessary. For example, the stability and linearity afforded by the invention enable large numbers of high gain carrier telephone repeaters (fifty or one hundred, or several hundred, for instance) to be operated in tandem, and thus render multiplex carrier telephony feasible over long circuits of high attenuation, as for example, nonloaded cable circuits one or several thousand miles long.

It is desired to point out that according to the invention and in contrast to prior art methods, the improvement realized in performance, for example in linearity, is not dependent upon a circuit balance of any sort nor upon constancy of balance or of some other relatively critical adjustment or condition, but depends upon the amount of gain reduction.

As shown hereinafter, feedback can be utilized according to this invention to improve the performance of the amplifier or the transducer in other respects also, as for example, with respect to phase shift and phase distortion. (For a discussion of phase distortion, reference may be had to the following publications: "Phase distortion in telephone apparatus"—C. E. Lane, Bell System Technical Journal, July 1930, pages 493–521; "Phase distortion and phase distortion correction" by S. P. Mead in the Bell System Technical Journal, vol. 7, No. 2 for April 1928 at page 195; "Building of sinusoidal currents in long periodically loaded lines" by J. R. Carson, in the Bell System Technical Journal, vol. 3, No. 4, at page 558.)

The feedback path may, in accordance with a feature of the invention, include wave shaping, adjusting or control devices for appropriately affecting the amplifier characteristic or performance. Thus, in addition to its primary function of feeding back a portion of the output waves in gain-reducing phase, it may feed back variable amounts of the output or components of different frequency in different degree or may change the phase or otherwise affect the properties or characteristics of the fed-back waves. As one example, the feedback path of a line repeater may contain a network of equivalent attenuation-versus-frequency characteristic to that of the line, to shape the repeater characteristic so as to compensate line distortion. This and other types of apparatus in the feedback path are disclosed.

The method of controlling the transmission characteristic of the amplifier by the transmission control networks in the feedback circuit is highly advantageous with respect to noise reduction. When the amplifier gain is reduced at any frequency by the feedback circuit, noise originating in either the forwardly transmitting path or the feedback path of the amplifier has its amplitude at the point of origin also reduced, at that frequency. Thus, for example, tube noises originating in any of the tubes of the amplifier (for instance, noises from Schottky effect or from microphonic action), noises from fluctuations of the voltages for energizing the tubes of the amplifiers, and resistance noise (thermal agitation) originating in the transmission control networks of the feedback path or in any other portion of the amplifier are reduced. Consequently, even at low gains, the signal-to-noise ratio in the load circuit is not in any way reduced.

The ability of the invention to improve the linearity and stability of an amplifier by use of negative feedback is of great economic as well as technical importance. The attainment of high power and high quality together in an amplifier has always been an object of especial desire since the power stage or stages in an amplifier are the most expensive to construct and operate. To make tubes in such stages larger so that a given output would represent a smaller load in comparison to their load capacity has been a costly expedient from both standpoints. The invention improves the characteristic of the power stage by adding gain at a lower power part of the amplifier, for example at the input, which can be done relatively cheaply, and by adding a negative feedback as already explained. Thus the same power stage can be operated with greatly improved characteristic or a much smaller power stage can be operated with equivalent quality of output.

Other objects and aspects of the invention will be apparent from the following description and claims.

In the drawings, Figs. 1, 5, 6, 7 and 8 are schematic diagrams of vacuum tube amplifier circuits used in explaining the invention;

Figs. 2, 3 and 4 are curves or plots for facilitating explanation of the invention, Fig. 3 being in the nature of an extension of Fig. 2, and Fig. 4 being a polar plot of functions plotted with rectilinear coordinates in Figs. 2 and 3;

Figs. 9, 10 and 11 are curves, and Figs. 12 and 13 vector diagrams, for facilitating explanation of the invention;

Figs. 14, 15 and 16 are symbolic representations of a simple feedback system for facilitating explanation of the invention; Fig. 17 shows an equivalent circuit of a simple feedback amplifier, illustrating application of principles explained by reference to Figs. 14 to 16; and Figs. 18, 19 and 20 illustrate application of methods of analysis explained by reference to Figs. 14 to 17;

Figs. 21, 22 and 24 are schematic circuit diagrams used in explaining the invention, especially with reference to control of modulation or distortion; and Fig. 23 is a vector diagram facilitating explanation of the invention, especially with relation to such control;

Figs. 25, 26, 27, 28, 29, and also Figs. 30, 31, 31A, 32, 32A, 33, 33A, and 34 are schematic circuit diagrams or equivalent circuits of vacuum tube amplifiers, used in explaining the invention, especially with reference to control of impedance in feedback systems;

Figs. 35A to 35E and 36A to 36H are schematic representations of various methods of feedback or configurations of feedback systems;

Fig. 37 is a diagrammatic showing of an amplifier circuit illustrative of multiple feedback in accordance with the invention;

Fig. 38 is a diagrammatic showing of an amplifier circuit illustrative of repetitions of the feedback process, in accordance with the invention;

Fig. 39 is a diagrammatic showing of a feedback amplifier circuit employing parallel transmission control networks in a feedback path;

Fig. 40 is a diagrammatic showing of a circuit illustrative of stabilization of a transmission loss by feedback, in accordance with the invention;

Figs. 41 and 42 are circuit diagrams for facilitating explanation of certain aspects of the invention; Fig. 43 shows a feedback amplifier embodying one form of the invention; Figs. 43A, 43B and 43C are diagrams, and Fig. 43D a set of curves for facilitating explanation of the operation of the amplifier of Fig. 43; and Fig. 43E shows a modification of the amplifier of Fig. 43;

Fig. 44 is a schematic diagram of a negative feedback amplifier embodying a form of the invention; Figs. 45 to 55 show curves facilitating explanation of the amplifier of Fig. 44; and Fig. 56 is a circuit diagram of the system shown schematically in Fig. 44;

Fig. 57 is a circuit diagram of a negative feedback amplifier suitable, for example, for amplifying waves of a frequency range extending from 4 kilocycles to 40 kilocycles in a non-loaded cable used for multiplex carrier telephony; and Figs. 58 to 64 show characteristics of the amplifier of Fig. 57;

Fig. 65 shows a circuit diagram of a negative feedback amplifier suitable, for example, for covering a frequency range from 8 kilocycles to 100 kilocycles, in multiplex carrier telephony over long non-loaded cables; and Fig. 65A shows a modification of the system of Fig. 65;

Fig. 66 is a circuit diagram of a negative feedback amplifier suitable, for example, for covering a frequency range from 3 kilocycles to 40 kilocycles, in multiplex carrier telephony over open-wire lines;

Fig. 67 shows gain-frequency characteristics of a high quality voice frequency amplifier employing negative feedback in accordance with the invention;

Fig. 68 shows curves plotted from measurements of second and third harmonics for an amplifier similar to that of Fig. 66;

Fig. 69 is a schematic circuit diagram of a single stage amplifier embodying a form of the invention with feedback through the output and input transformers; and Fig. 70 shows gain-frequency characteristics of the amplifier of Fig. 69;

Fig. 71 shows gain-frequency characteristics of the amplifier Fig. 65;

Fig. 72 is a circuit diagram of a two-stage amplifier embodying a form of the invention, with feedback through the output transformer;

Fig. 73 is a schematic circuit diagram of an amplifier with negative feedback reducing distortion and stabilizing gain and with a second feedback action giving transmission equalization; and

Figs. 74 and 75 show amplifiers embodying forms of the invention employing positive feedback; and Fig. 75A shows a modification of the system of Fig. 75.

I. Amplification, General Considerations

I. a. Amplification with and without feedback

To derive general formulae in what is thought to be the simplest possible manner, two assumptions will be made; first, that the application of feedback does not give rise to instability or electrical oscillations; and second, that the principle of superposition is applicable. (The principle of superposition is the principle which states that the instantaneous current which flows at any point in a network, under the simultaneous action of a number of electromotive forces impressed anywhere in the circuit containing the network, is the algebraic sum of the instantaneous currents which would flow at the same point if the various electromotive forces acted independently.) This procedure compels determining separately the conditions for oscillation and non-oscillation and naturally requires justifying the second assumption. Mr. H. Nyquist has shown that neither assumption is necessary to a general investigation of the operation of the circuit. ("Regeneration Theory"—H. Nyquist, The Bell System Technical Journal, Vol. XI, January 1932, pages 126–147.)

Referring to Fig. 1, which shows the equivalent circuit of a simple feedback amplifier, assume that the amplifier is excited by a sinusoidal signal voltage, E, in series with an impedance C. All symbols are vectors or complex quantities unless otherwise stated and approximate steady state conditions only are evaluated. The performance will be considered without regard to harmonics, the subject of modulation and harmonic production being presented separately. If the foregoing assumption regarding the absence of self-oscillations or instability has been satisfied, then evidently there will be a voltage, V, across the grid of signal frequency and under the control of E; the first step is to evaluate V.

The application of a voltage, V, to the grid of the first tube results in power being released in the plate circuit of that tube and all succeeding tubes. The plate or internal resistance of the last tube is designated R_0. Referring to Fig. 1, μV is a fictitious voltage in series with R_0. This defines μ: μ is a complex quantity such that when multiplied by V it makes the product μV when interpreted as a fictitious voltage acting in series with R_0 give rise to a current in the output identical in phase and amplitude with the actual current in the output due to the actual alternating voltage, V, on the grid. This is the justification for the equivalent circuit in Fig. 1. The extent to which it may or may not be equivalent will be taken up in connection with the study of modulation. If it is equivalent, then evidently the principle of superposition may be applied and, furthermore, its application is not essential but merely convenient.

For convenience a table is given at this point showing symbols and derived equations for amplification with and without feedback.

Table—(All symbols are complex quantities or vectors)

E=input signal
G_0=grid impedance
V=final voltage effective on the grid
V_1=applied voltage from the input
V_2=feedback voltage
By principle of superposition
V_1 is due to E with μV shorted
V_2 is due to μV with E shorted
$V=V_1+V_2$
$V_2=\mu\beta V$, defining β
$V=V_1+\mu\beta V$

$$V=\frac{V_1}{1-\mu\beta}=\frac{\text{voltage without feedback}}{1-\mu\beta}$$

$$\mu V=\frac{\mu}{1-\mu\beta}V_1=\text{plate circuit driving voltage with feedback}$$

μV_1=plate circuit driving voltage without feedback

$$V_2=\frac{\mu\beta}{1-\mu\beta}V_1=\text{feedback voltage}$$

If

$$\mu\beta>>1,\ \mu V\doteq-\frac{1}{\beta}V_1$$

showing that in this case the amplification approaches

$$-\left(\frac{1}{\beta}\right)$$

which is independent of μ.

If μ is the amplification without feedback,

$$\frac{\mu}{1-\mu\beta}$$

is the amplification with feedback and

$$\frac{1}{1-\mu\beta}$$

is the change in amplification due to feedback.

Applying the principle of superposition, V is equated to (V_1+V_2) and is to be thought of as the sum of two voltages; first, V_1, the applied grid voltage due to the generator voltage, E; and second, V_2, the feedback voltage across the grid due to μV or in other words to the fact that the output is connected to the input.

$$V_1=\frac{\dfrac{G_0\left(f+\dfrac{R_0L}{R_0+L}\right)}{G_0+f+\dfrac{R_0L}{R_0+L}}}{C+\dfrac{G_0\left(f+\dfrac{R_0L}{R_0+L}\right)}{G_0+f+\dfrac{R_0L}{R_0+L}}}E \qquad (a)$$

Equation (a) follows directly from the configuration of the equivalent circuit in Fig. 1 and is special to that particular configuration; it has been written to illustrate the meaning of V_1. This expression is obtained by observing that voltage E works into impedance C in series with a composite impedance of the form given in the numerator of Equation (a). This in turn is obtained by taking the individual impedances in proper relation, referring to the circuit configuration of Fig. 1, in accordance with well-known circuit theory.

$V_2=\mu\beta V$ defines β: β is the complex quantity by which μV, the driving voltage in series with R_0, must be multiplied to obtain the voltage, V_2, that is—the driving voltage alone—will produce across the grid, G_0.

$$\beta=\frac{\dfrac{CG_0}{C+G_0}}{\dfrac{R_0L}{R_0+L}+f+\dfrac{CG_0}{C+G_0}}\left(\frac{L}{R_0+L}\right) \qquad (b)$$

This expression is obtained similarly to Equation (a) by considering in Fig. 1 that E is absent and writing as an expression of impedances the fraction of the voltage of the fictitious generator in the space path of the tube, that will be impressed back on the grid.

Equation (b) also applies only to the circuit in Fig. 1 and has been written to illustrate the meaning of β.

The foregoing definitions of μ and β should be kept in mind because it will appear later that the μ and β-circuits have utterly different properties. It is for this reason that the quantity by which a voltage is multiplied in traversing the closed path once has been designated $\mu\beta$, the product of two complex quantities, rather than by a single symbol.

Referring again to Fig. 1, the current in the load impedance L is proportional to the driving voltage in series with R_0. Therefore, a comparison of the driving voltages with and without feedback will be a comparison of the amplifications with and without feedback. With feedback, this voltage is

$$\frac{\mu}{1-\mu\beta}V_1$$

Without feedback, this voltage is to be taken as μV_1 by definition. The reason for this definition, which may appear arbitrary, will become apparent as additional equations are derived because of the simplifications that it leads to; in the case of other arrangements, as for example in Fig. 5 where the feedback circuit is conjugate to both the input and output circuits, the definition is in accord with experience if the "without feedback" condition is obtained by opening the feedback connection, f, or by shorting f to the cathode. With this convention in mind, μ is the amplification without feedback;

$$\frac{\mu}{1-\mu\beta}$$

the amplification with feedback; and

$$\frac{1}{1-\mu\beta}$$

the change in amplification due to feedback.

Referring to Fig. 1, if $\mu\beta$ is large compared to unity $(1-\mu\beta)\doteq-\mu\beta$ and the amplification with feedback approaches

$$\frac{-1}{\beta}$$

or is largely independent of the amplification or variations in amplification of the tubes. In this case a db loss in the β-circuit raises the gain of the amplifier by a corresponding amount and vice-versa. Since

$$\frac{\mu}{1-\mu\beta}$$

is the amplification; Θ, the argument of the complex number symbolized by this quotient represents the phase shift from the grid of the first tube to the plate of the last tube or more specifically, the phase shift of μV relative to V_1. If $\mu\beta>>1$ so that the amplification approaches

$$-\frac{1}{\beta}$$

and if at the same time the phase shift in the β-circuit is zero, meaning that β is a scalar quantity; then Θ, the phase shift of the amplifier, approaches 180° irrespective of the phase shift in the μ-circuit or the number of tubes. This differs from the usual type of amplifier without feedback where the phase shift is changed 180° by the addition of each tube.

I. b. Feedback

In considering the operation of feedback arrangements of which the circuit in Fig. 1 is a particular example, the following three terms evidently relate to feedback:

$$\beta;\ \frac{\mu\beta}{1-\mu\beta};\ \text{and}\ \frac{1}{1-\mu\beta}$$

The quantity $\mu\beta$ is a complex quantity representing the ratio by which the amplifier and feedback circuit (or more generally, the μ and β) modify a voltage in a single journey around the closed path.

$$\frac{\mu\beta}{1-\mu\beta}$$

is the quantity by which a voltage without feedback is multiplied to get the feedback voltage. To illustrate this, in Fig. 1 the grid voltage, for example, is V_1 without feedback whereas

$$\left(V_1+\frac{\mu\beta}{1-\mu\beta}V_1\right)$$

is the grid voltage with feedback and accordingly it is convenient to refer to

$$\frac{\mu\beta}{1-\mu\beta}V_1$$

as the feedback voltage itself at this place in the circuit (the grid) inasmuch as without feedback this voltage is not present and with feedback it is. Lastly,

$$\frac{1}{1-\mu\beta}$$

is a complex quantity representing the ratio of the voltage with feedback to the voltage without feedback or the voltage with feedback is the voltage without feedback divided by $(1-\mu\beta)$. These three relationships are dependent and if one of them is known the other two can be determined.

Herein

$$\frac{1}{1-\mu\beta}$$

is used as a quantitative measure of the amount of feedback and the feedback is described as positive feedback or negative feedback according as the absolute value of

$$\frac{1}{1-\mu\beta}$$

is greater or less than unity. For the purpose of this specification the term "feedback" is not limited to merely those cases where the value of

$$\frac{1}{1-\mu\beta}$$

is other than unity.

I. c. Criterion for stability and instability against oscillating or "singing"

Fig. 2 shows curves giving the phase shift around the feedback path plotted as a function of the absolute value of $\mu\beta$. This figure includes a family of curves in which the db change in gain due to feedback is the parameter. It also includes a set of boundary curves A, B, C, D, E, F, G, H and I which gives either limiting or significant values of Φ and absolute values of $\mu\beta$ with regard to gain and gain stability.

This chart shows the effect of feedback upon the gain of the amplifier and illustrates that the result depends upon the absolute value as well as the argument of $\mu\beta$. One striking feature about the chart is that it implies that even though the phase shift is 360° and the absolute value of $\mu\beta$ exceeds unity, oscillations will not result. This may or may not be true. On first thought it might appear that owing to practical non-linearity, oscillations would result whenever the gain around the closed circuit was equal to or greater than the loss and simultaneously the phase shift was zero or in other words

$$\mu\beta=|\mu\beta|+j0\geqslant 1$$

With this in mind applicant constructed a three tube amplifier where at 36 kc.,

$$\mu\beta=100\underline{|330^\circ}$$

This amplifier was stable and its gain was reduced 40 db due to feedback. At the same time a one tube circuit was available where at 10 kc.,

$$\mu\beta=9\underline{|270^\circ}$$

and this circuit was always self-oscillating at about 10 kc. although

$$\mu\beta=|\mu\beta|+j0\geqslant 1$$

at a higher frequency. The article "Regeneration theory" by H. Nyquist, referred to above, published in Bell System Technical Journal for January 1932 pages 126 to 147 discusses this question of oscillation or non-oscillation and gives a criterion for stability.

A practical way of using this criterion is to plot $\mu\beta$ in polar coordinates (the amplitude and angle of $\mu\beta$ vary with frequency) for all values of frequency from $-\infty$ to $+\infty$. If the resulting loop or loops enclose the point (1,0) the system is unstable; if the point (1,0) is not enclosed the system is stable. (Reference may be made to the Nyquist paper cited for a discussion of what is meant by enclosing the point (1,0).)

The values of $\mu\beta$ as a function of frequency for the amplifier circuit shown in Fig. 57 (described hereinafter) have been measured experimentally and are plotted in the form of such a polar diagram in Fig. 58. It will be noticed that the point (1,0) is not included by the resulting loop and hence it is to be inferred in accordance with the foregoing rule that such an amplifier would be stable. This was found to be the case and the measurements submitted in Figs. 59, 60, 61 and 62 were all made on this particular amplifier. Fig. 58 may be viewed as a graph of

$$\mu\beta=|\mu\beta|\underline{|\Phi}$$

where $\mu\beta=F(\mu\beta,+f)$ and f is the frequency. The branch $\rho=F(\mu\beta,-f)$ required according to the rule although not shown in Fig. 58 could be taken as the conjugate of $\mu\beta=F(\mu\beta,+f)$. In many instances, only positive values of frequency need to be considered.

I. d. Relationship between feedback and amplification

Referring again to Fig. 2, it will be noticed that one of the parametric contour lines, line A, is the locus of zero change in gain. Along this contour, for varying values of $\mu\beta$ ranging from 0–2, the gain does not change provided the phase shift around the μ and β-circuit equals

$$\cos^{-1}\frac{|\mu\beta|}{2}$$

This in turn requires that Φ be in either the first or fourth quadrants, that is, $-90°\leqslant\Phi\leqslant+90°$. All conditions above the contour line

$$\Phi=\cos^{-1}\frac{|\mu\beta|}{2}$$

are those of positive feedback or an increase in gain and in general result in a degradation of transmission, at least as regards linearity; conditions below the boundary correspond to negative feedback or a reduction in gain and are accompanied by an improvement both with respect to linearity and stability of amplification.

Considering values above the boundary contour A, if Φ is restricted to values between $-90°$ to $+90°$ ($\Phi=\cos^{-1}|\mu\beta|$ excepted) then for each positive and negative value of Φ there are two different magnitudes of $|\mu\beta|$ that will produce the same increase in gain, for example, 1 db. For the special case of $\Phi=\cos^{-1}|\mu\beta|$, there is, for each positive and negative value of Φ, only one value of $|\mu\beta|$ that will produce a particular increase in gain with positive feedback, and this increase is the maximum possible for that particular value of Φ. This suggests a method for measuring Φ. Considering values below the boundary A, as Φ varies from 0–360°, there is, for each positive and negative value of Φ, only one value of $|\mu\beta|$ that will produce a particular decrease in gain, e. g. 1 db.

Studying Fig. 2 further, in the case of positive feedback, the maximum phase shift around the μ and β-circuit for a particular increase in gain is given by $\Phi_{max.}=\cos^{-1}|\mu\beta|$. The contour lines in this case fall on either side of the locus of $\cos^{-1}|\mu\beta|$. Thus, it is clear from Fig. 2 that for very large increases in gain, $\mu\beta \doteq 1$. As already has been indicated, with substantial amounts of negative feedback, the over-all amplification in db. approaches the β-circuit loss in db. and the phase shift offered by the feedback circuit to the transmitted wave is that of the β-circuit reversed in sign and rotated 180° and this happens whether the number of tubes is even or odd.

With amplifiers having positive feedback, in accordance with the invention, that feedback can be made to improve the stability while increasing the gain. The gain stability of an amplifier having positive feedback may be improved, degraded, or not changed as compared to the stability without feedback depending upon, first, the amount of feedback and, second, the phase shift encountered in a single journey around the amplifier and feedback circuits. In general, the distortion due to modulation in an amplifier is not reduced by positive feedback.

Fig. 3 is an extension of Fig. 2 plotted on a logarithmic scale to include larger values of $|\mu\beta|$. Thus, as in Fig. 2, there is a set of boundary curves indicated as A, B, C, D, E, F, G, H and I which gives either limiting or significant values of $|\mu\beta|$ and Φ with regard to gain and gain stability, and there is also a family of curves in which db change in gain due to feedback is the parameter.

Fig. 4 is a polar diagram of the functions plotted in Figs. 2 and 3 including also a set of boundary curves A, B, C, etc., corresponding to those of Figs. 2 and 3, and a family of db-change curves corresponding to those of Figs. 2 and 3. It is interesting to note that the locus of the contour lines of constant db change in gain are a family of concentric circles. This may be shown as follows:—

Let

$$F = \frac{1}{1-\mu\beta}$$

be the change in amplification due to feedback, and G_{CF}, the db change in grain due to feedback

$$G_{CF} = 20 \log_{10} \frac{1}{|1-\mu\beta|}$$

$$G_{CF} = 20 \log_{10} \frac{1}{\sqrt{1-2\,|\mu\beta| \cos \Phi + |\mu\beta|^2}}$$

$$G_{CF} = -10 \log_{10} (1-2\,|\mu\beta| \cos \Phi + |\mu\beta|^2)$$

$$10^{-\frac{G_{CF}}{10}} = 1-2\,|\mu\beta| \cos \Phi + |\mu\beta|^2$$

which is a family of concentric circles of radius

$$10^{-\frac{G_{CF}}{10}}$$

about the point 1,0.

I. e. Applications of balanced bridges to make feedback path conjugate to input and output connecting circuits

In Fig. 5 is shown an arrangement convenient for some purposes where, by using balanced Wheatstone bridge networks in both input and output circuits, the feedback is conjugate to both. For example, assuming the output bridge is balanced, the voltage fed back will be independent of the value of L. Inasmuch as the input bridge is similarly arranged and balanced, the proportion of the voltage fed back and applied to the input bridge that appears across the grid will be

$$\frac{G_0}{G+G_0}$$

and this proportion will be independent of C. In this figure all symbols are complex quantities or vectors. The circuit of this figure may comprise any number of stages, and this is indicated on the drawings by showing the grid of the first tube and the plate of the last tube, both of which tubes may, of course, be one and the same. This is likewise true of other figures such as Figs. 1, 6, 7 and 8 which have a grid and a plate similarly labeled.

By the use of balanced bridge networks as shown in Fig. 5, feedback, in addition to being made independent of the impedances of the connecting circuits, can be more easily modified and controlled. Moreover, the input and output impedances of the feedback amplifier which are joined to the connecting circuits are unaffected by the amount of feedback. Similarly, if networks are inserted in the β-circuit, the impedances which these networks face are independent of the impedances of the input and output connecting circuits, C and L. In Fig. 5, it follows from Kirchoff's laws, that

$$V_1 = \frac{\dfrac{G_0 G}{G+G_0}}{C+(1+K')\dfrac{G_0 G}{G+G_0}} E \qquad (c)$$

In Fig. 5, if the β-circuit network shown is omitted, it follows from Kirchoff's laws, that

$$\beta = \frac{\dfrac{G_0}{G+G_0}}{\dfrac{1+K}{K} + \left(\dfrac{1+K'}{K'}\right)\dfrac{R+R_0}{G+G_0}} \qquad (d)$$

Equations (c) and (d) apply to the circuit in Fig. 5 and are written to illustrate further the meaning of V_1 and β.

A fuller treatment of the subject of input and output bridges is given at a later point under section V. e.

II. Stability

II. a. Stability of amplification with respect to variations in μ and β

In considering stability, it will be helpful to distinguish between amplification, amplification ratio, and gain. As used in this specification, these terms have the following significance:

Amplification of an amplifier without feedback is defined as the quantity by which the voltage across the grid of the first tube must be multiplied to obtain the phase and magnitude of an equivalent generator in series with the plate resistance (R_0) of the last tube. The measure of amplification is noted as μ, a complex quantity.

Amplification ratio of an amplifier without feedback is the modulus of the complex quantity noted as μ. It may be obtained by taking the square-root of the sum of the squares of the real and imaginary components of the complex quantity. The notation used is $|\mu|$.

It is to be noted that μ is a complex quantity, the complexed measure of a voltage coefficient, which gives the magnitude of the ratio of the voltage and the shift in phase between these two points.

As indicated hereinbefore, the gain is twenty times the logarithm of the amplification ratio.

In Fig. 1 it was shown that the amplification with feedback is

$$\frac{\mu}{1-\mu\beta}$$

In thinking of the effect of feedback upon stability it will be convenient at first to view stability of amplification as a change, ∂A_F, in A_F (where A_F is the amplification with feedback) due to a change in either μ or β and these effects may be derived by assuming that the variations are small and then replacing μ or β by $(\mu+\partial\mu)$ or $(\beta+\partial\beta)$.

This derivation could be made on the basis of the Fig. 1 circuit. It is not limited to any particular type of circuit, however, but is perfectly general. While no attempt is made, therefore, to illustrate all types of feedback circuits, it may be of interest in following the derivation to have in mind concrete examples, some of which are given in Figs. 6, 7, and 8. In Fig. 6 the feedback is from a point on an output bridge to the grid through the secondary of an input transformer. In Fig. 7 the feedback is from an output bridge through a transformer to an input bridge. In Fig. 8 the feedback is from the output to an input bridge.

In any of these circuits, or in general, the derivations are as follows, considering first stability of amplification with respect to small changes in μ and β:

$$A_F=\frac{\mu}{1-\mu\beta}=\text{amplification with feedback}$$

(Showing as in Fig. 1 that the amplification is increased or decreased according as the feedback is positive or negative.) Taking the logarithms and then the partial differentials gives at once the ratio of ∂A_F to A_F, or (multiplied by 100) the percentage change in A_F due to a percentage change in μ.

$$\log A_F=\log \mu-\log(1-\mu\beta)$$

$$\left[\frac{\partial A_F}{A_F}\right]_\mu=\frac{\partial\mu}{\mu}-\frac{-\beta\partial\mu}{1-\mu\beta}=\frac{\dfrac{\partial\mu}{\mu}}{1-\mu\beta}$$

which demonstrates that for small variations, the stability is worse or better according as the feedback is positive or negative.

Similarly

$$\left[\frac{\partial A_F}{A_F}\right]_\beta=\frac{\mu\beta}{1-\mu\beta}\left[\frac{\partial\beta}{\beta}\right]$$

If

$$\mu\beta>>1;\ \left[\frac{\partial A_F}{A_F}\right]_\mu=\frac{\dfrac{\partial\mu}{\mu}}{-\mu\beta}$$

and

$$\left[\frac{\partial A_F}{A_F}\right]_\beta=-\frac{\partial\beta}{\beta}$$

From the above it is seen that the ratio of ∂A_F to A_F due to a change in μ equals the ratio of $\partial\mu$ to μ divided by $(1-\mu\beta)$; on the other hand, the ratio of ∂A_F to A_F due to a change in β equals

$$\left(\frac{\mu\beta}{1-\mu\beta}\right)$$

multiplied by the ratio of $\partial\beta$ to β. From these relationships, it is also deduced that if $(\mu\beta>>1)$; μ or the μ-circuit is stabilized by an amount at least corresponding to the reduction in amplification and, hence, the effect of introducing a gain or loss in the μ-circuit is to produce no material change in the over-all amplification of the system; the stability of amplification as affected by β or the β-circuit is neither appreciably improved or degraded and the introduction of a loss in the β-circuit raises the gain of the amplifier by an amount almost corresponding to the loss introduced and vice-versa.

If μ and β are both varied and the variations are sufficiently small, the net effect is the same as if each were changed separately and the two results then combined. This is indicated by differential Equation (e).

$$\left.\frac{\partial A_F}{A_F}\right\}_{\mu\beta}=\frac{\dfrac{\partial\mu}{\mu}}{1-\mu\beta}+\frac{\mu\beta}{1-\mu\beta}\frac{\partial\beta}{\beta} \qquad (e)$$

In other practical instances where the effect of positive feedback is to increase the amplification by a sufficiently large amount or in other cases where the changes in μ and β are not sufficiently small, then the above formulae for amplification stability do not apply. To derive expressions for these larger variations, let

$$A_F=\frac{\mu}{1-\mu\beta}$$

be the amplification before making the change and

$$A_F+\Delta A_F=\frac{\mu+\Delta\mu}{1-(\mu+\Delta\mu)(\beta+\Delta\beta)}$$

represent the amplification after making the change and assume no restrictions as to the way or amount of variation in μ or β, denoted $\Delta\mu$ and $\Delta\beta$ respectively.

$$\Delta A_F=\left\{\begin{array}{l}\dfrac{\mu}{1-\mu\beta-\mu\Delta\beta-\beta\Delta\mu-\Delta\mu\Delta\beta}+\\ \qquad\qquad\dfrac{\Delta\mu}{1-(\mu+\Delta\mu)(\beta+\Delta\beta)}\\ -\dfrac{\mu}{1-\mu\beta}\end{array}\right.$$

$$\Delta A_F=\left\{\begin{array}{l}\dfrac{\mu}{1-\mu\beta}\,\dfrac{\dfrac{\Delta\mu}{\mu}}{1-(\mu+\Delta\mu)(\beta+\Delta\beta)}\\ +\dfrac{\mu}{1-\mu\beta}\left(1+\dfrac{\Delta\mu}{\mu}\right)\dfrac{\mu\beta}{1-(\mu+\Delta\mu)(\beta+\Delta\beta)}\left(\dfrac{\Delta\beta}{\beta}\right)\end{array}\right.$$

$$\left.\frac{\Delta A_F}{A_F}\right\}_{\mu\beta}=\left\{\begin{array}{l}\dfrac{\dfrac{\Delta\mu}{\mu}}{1-(\mu+\Delta\mu)(\beta+\Delta\beta)}\\ +\left(1+\dfrac{\Delta\mu}{\mu}\right)\dfrac{\mu\beta}{1-(\mu+\Delta\mu)(\beta+\Delta\beta)}\left(\dfrac{\Delta\beta}{\beta}\right)\end{array}\right. \qquad (f)$$

Equation (f) is the ratio of the change in amplification, ΔA_F to the original amplification A_F, when μ and β are replaced by $(\mu+\Delta\mu)$ and $(\beta+\Delta\beta)$. To get the effect of varying μ alone, replace

$$\frac{\Delta\beta}{\beta}$$

in (f) by zero; if β changes and μ does not, replace

$$\frac{\Delta\mu}{\mu}$$

in (f) by zero. This leads to Equations (g) and (h), respectively.

$$\left.\frac{\Delta A_F}{A_F}\right\}_\mu=\frac{\dfrac{\Delta\mu}{\mu}}{1-(\mu+\Delta\mu)\beta} \qquad (g)$$

$$\left.\frac{\Delta A_F}{A_F}\right\}_\beta=\frac{\mu\beta}{1-\mu(\beta+\Delta\beta)}\left(\frac{\Delta\beta}{\beta}\right) \qquad (h)$$

Replacing

$$\frac{\Delta\mu}{\mu} \text{ by } \frac{\partial\mu}{\mu}$$

$$\frac{\Delta\beta}{\beta} \text{ by } \frac{\partial\beta}{\beta}$$

and

$$\frac{\Delta A_F}{A_F} \text{ by } \frac{\partial A_F}{A_F}$$

or

$$\frac{dA_F}{A_F}$$

(f), (g) and (h) lead to (i), (j) and (k) as previously derived.

$$\left.\frac{\partial A_F}{A_F}\right\}_{\mu\beta}=\frac{\frac{\partial\mu}{\mu}}{1-\mu\beta}+\frac{\mu\beta}{1-\mu\beta}\,\frac{\partial\beta}{\beta} \qquad (i)$$

$$\left.\frac{\partial A_F}{A_F}\right\}_\mu=\frac{\frac{\partial\mu}{\mu}}{1-\mu\beta} \qquad (j)$$

$$\left.\frac{\partial A_F}{A_F}\right\}_\beta=\frac{\mu\beta}{1-\mu\beta}\,\frac{\partial\beta}{\beta} \qquad (k)$$

In applying these results it should be kept in mind that the functions involved are functions of complex variables and terms of the form

$$\frac{\partial\mu}{\mu}$$

for example, are symbols for operations that have not been performed. As an illustration, consider

$$A_F=\frac{\mu}{1-\mu\beta}$$

which approaches

$$\left(\frac{1}{-\beta}\right)$$

if $\mu\beta>>1$. Assuming $|\mu\beta|=10$, the approach of A_F to

$$\left(\frac{1}{-\beta}\right)$$

will be about 10 times as precise if the argument of $\mu\beta$ is 90° as when it is 0° or 180°. In this respect, 20 db negative feedback when the argument of $\mu\beta$ is 90° is approximately equivalent in its effect on A to a negative feedback of 40 db with an argument of $\mu\beta$ of 0° or 180°. This illustration is of practical significance where equalizers are inserted in the β-circuit and it is desired to control the equalization very accurately with respect to variations in μ.

II. b. *Stability of gain with respect to variations in μ, β and Φ*

In many practical applications of amplifiers it is the change in gain or the change in ammeter or voltmeter reading at the output that is a measure of the stability rather than the stability of amplification (vector ratio) as treated in the preceding section. The conditions surrounding the gain stability may be examined by considering the absolute value of the amplification. This may be shown as follows: Since

$$A_F=\frac{\mu}{1-\mu\beta}$$

where

$$\mu\beta=|\mu\beta|\,\underline{|\Phi},$$

and

$$\mu=|\mu|\,\underline{|\Theta}$$

$$A_F=\left|\frac{\mu}{\sqrt{1-2\,|\mu\beta|\cos\Phi+|\mu\beta|^2}}\right|\underline{\left|\tan^{-1}\frac{\sin\Theta+|\mu\beta|\sin(\Phi-\Theta)}{\cos\Theta-|\mu\beta|\cos(\Phi-\Theta)}\right.}$$

$$|A_F|=\frac{|\mu|}{\sqrt{1-2\,|\mu\beta|\cos\Phi+|\mu\beta|^2}}$$

Taking the logarithm to the base ϵ of both sides and differentiating partially, it may be shown that:—

$$\left[\frac{\partial|A_F|}{|A_F|}\right]_{|\mu|}=\frac{\frac{\partial|\mu|}{|\mu|}}{|1-\mu\beta|^2}[1-|\mu\beta|\cos\Phi] \qquad (l)$$

(That

$$\left[\frac{\partial|A_F|}{|A_F|}\right]_{|\mu|}$$

is also indicative of the corresponding db change in gain, $\partial G_F)|\mu|$, follows from the relationship

$$G_F=20\log_{10}|A_F|$$

whence

$$G_F=8.686\,ln|A_F|$$

and

$$\partial G_F)|\mu|=8.686\left[\frac{\partial|A_F|}{|A_F|}\right]_{|\mu|}$$

where G_F is used to indicate gain with feedback.)

It can be seen from (l) that it is possible to choose $\mu\beta$ so that the stability is perfect, assuming that the variations are sufficiently small. To make the absolute value or magnitude of the amplification independent of small variations in μ, $[1-|\mu\beta|\cos\Phi]=0$ or $\Phi=\cos^{-1}|\mu\beta|^{-1}$.

Referring again to Figs. 2, 3 and 4, the boundary C is the locus of $\Phi=\cos^{-1}|\mu\beta|^{-1}$ and hence includes all the perfect amplifiers from the standpoint of gain stability with respect to small variations in $|\mu|$. It will be observed in this regard that with respect to gain, it is possible to stabilize an amplifier where the feedback is such as to lead to positive regeneration. In other words, regeneration may be utilized to raise the gain of the amplifier and yet at the same time the gain stability of the amplifier with feedback need not be degraded but on the contrary may be improved. If a similar procedure is followed for the case of an amplifier having negative feedback, then the stabilization in gain obtained is theoretically perfect and independent of the reduction in gain due to feedback.

To accomplish this end, $|\mu\beta|$ and Φ should be so related that $\Phi=\cos^{-1}|\mu\beta|^{-1}$. In this connection, it should be noted that, inasmuch as the cosine of an angle can not exceed unity, $|\mu\beta|\geqslant 1$ and hence $|\mu\beta|$ lies to the right of boundary D in Fig. 2 and, accordingly, in view of the equation for boundary C, both positive and negative feedback amplifiers are able to possess this extra stabilizing property.

It should also be observed that inasmuch as the absolute value of a complex quantity is never negative, $-90°\leqslant\Phi\leqslant+90°$ for all amplifiers designed in accordance with the rule $\Phi=\cos^{-1}|\mu\beta|^{-1}$. For large values of negative feedback ($\mu\beta>>1$) this would require that Φ approach but never equal $\pm 90°$. In Figs. 2 and 4, C approaches E asymptotically.

As may be deduced from (l), if $\Phi=\pm 90°$, the gain stability, instead of being perfect, is improved by an amount corresponding to twice the db reduction in gain due to feedback. For example, if

$$\mu\beta = 1000\underline{|\pm 90^\circ}$$

the variation in amplification

$$\left(\frac{\partial A_F}{A_F}\right)_\mu$$

would be improved a thousandfold and

$$\left[\frac{\partial|A_F|}{|A_F|}\right]_{|\mu|}$$

which is a measure of the gain stability, would be improved a millionfold. On the other hand, if

$$\mu\beta = 1000\underline{|\pm 89^\circ - 56' - 56''}$$

the gain stability would theoretically be perfect instead of being improved a millionfold

$$(\Phi = \pm 90^\circ)$$

whereas the improvement in stability of amplification,

$$\left[\frac{\partial A_F}{A_F}\right]$$

would not be appreciably affected, its improvement being a thousandfold as before.

Referring again to Fig. 2, the boundary, B, which is the locus of $\Phi = \cos^{-1}|\mu\beta|$ defines, for all feedback amplifiers, the relationship between $|\mu\beta|$ and Φ in order that the change in gain due to feedback be independent of small variations in $|\mu|$ assuming β and Φ constant. An amplifier fulfilling this condition would be one whose gain stability with feedback was the same as without feedback. That $\Phi = \cos^{-1}|\mu\beta|$ gives this result may be proven by substituting $|\mu\beta|$ for $\cos\Phi$ in (l) or may be shown as follows:

Let F=change in amplification due to feedback

$$F = \frac{1}{1-\mu\beta} \text{ where } \mu\beta = |\mu\beta|\underline{|\Phi}$$

$$|F| = \frac{1}{\sqrt{1 - 2|\mu\beta|\cos\Phi + |\mu\beta|^2}}$$

Taking the logarithm to the base ϵ of both sides and differentiating partially:—

$$\left[\frac{\partial|F|}{|F|}\right]_{|\mu|} = \frac{|\mu\beta|}{|1-\mu\beta|}\frac{\frac{\partial|\mu|}{|\mu|}}{|1-\mu\beta|}[\cos\Phi - |\mu\beta|] \qquad (m)$$

Substituting $\Phi = \cos^{-1}|\mu\beta|$ in (m) gives

$$\frac{\partial|F|)}{|F|)|_{|\mu|}} = 0$$

or that the variation in the change of gain due to feedback is independent of small variations in $|\mu|$. Consequently the gain with feedback varies, due to small variations in $|\mu|$ exactly as it does without feedback as may be shown by considering that

$$|A_F| = |\mu|\ |F| \text{ where } |F| = \frac{1}{\sqrt{1-|\mu\beta|\cos\Phi + |\mu\beta|^2}}$$

and noting that

$$\frac{\partial|F|)}{|F|)|_{|\mu|}} = \frac{\partial|\mu|}{|\mu|} + \frac{\partial|F|}{|F|}$$

Referring to Fig. 2 and recapitulating, with respect to the gain stability of the feedback amplifier as affected by small variations in $|\mu|$, the situation is indicated by Equation (l) and may be described as follows:—

II. c. Gain stability of feedback amplifiers with respect to small variations in $|\mu|$

General equation.

$$\left[\frac{\partial|A_F|}{|A_F|}\right]_{|\mu|} = \frac{\frac{\partial|\mu|}{|\mu|}}{1-2|\mu\beta|\cos\Phi + |\mu\beta|^2}[1-|\mu\beta|\cos\Phi] \quad (la)$$

Boundary G.—Stability is improved by an amount corresponding to the reduction in gain due to feedback

$$\left[\frac{\partial|A_F|}{|A_F|}\right]_{|\mu|} = \frac{\frac{\partial|\mu|}{|\mu|}}{|1-\mu\beta|^2}[1-|\mu\beta|\cos\Phi] = \frac{\frac{\partial|\mu|}{|\mu|}}{1+|\mu\beta|}$$

obtained by putting $\Phi = \pm 180^\circ$. The feedback must be negative; it can not be positive and meet this condition.

$$|\mu\beta| \geqq 0$$

Boundary E.—Stability improved by an amount corresponding to twice the reduction in gain due to feedback

$$\left[\frac{\partial|A_F|}{|A_F|}\right]_{|\mu|} = \frac{\frac{\partial|\mu|}{|\mu|}}{|1-\mu\beta|^2}[1-|\mu\beta|\cos\Phi] = \frac{\frac{\partial|\mu|}{|\mu|}}{1+|\mu\beta|^2}$$

obtained by putting $\Phi = \pm 90^\circ$. The feedback must be negative; it can not be positive and meet this condition.

$$|\mu\beta| \geqq 0$$

Boundary B.—Stability unaffected by feedback

$$\left[\frac{\partial|A_F|}{|A_F|}\right]_{|\mu|} = \frac{\partial|\mu|}{|\mu|} \text{ obtained by putting } \cos\Phi = |\mu\beta|$$

$$\sqrt{1-2|\mu\beta|\cos\Phi + |\mu\beta|^2} = \sqrt{1-|\mu\beta|^2}$$

The feedback must be positive; it can not be negative and meet this condition.

$$0 \leqslant |\mu\beta| \leqslant 1$$

Boundary D.—Stability with feedback twice as good as stability without feedback

$$\left[\frac{\partial|A_F|}{|A_F|}\right]_{|\mu|} = \frac{1}{2}\frac{\partial|\mu|}{|\mu|} \text{ obtained by putting } |\mu\beta| = 1$$

$$|1-\mu\beta| = \sqrt{2}\sqrt{1-\cos\Phi}$$

The feedback may be either positive or negative and meet this condition.

Boundary C.—Perfect amplifier stability

$$\left[\frac{\partial|A_F|}{|A_F|}\right]_{|\mu|} = 0 \text{ obtained by putting } \cos\Phi = \frac{1}{|\mu\beta|}$$

The feedback may be either positive or negative and still meet this condition.

$$|\mu\beta| \geqslant 1$$

Boundary I.—Stability improved by an amount corresponding to twice the reduction in gain due to feedback and reversed in sign

$$\left[\frac{\partial|A_F|}{|A_F|}\right]_{|\mu|} = \frac{\frac{\partial|\mu|}{|\mu|}}{|1-\mu\beta|^2}[1-|\mu\beta|\cos\Phi] = \frac{-\frac{\partial|\mu|}{|\mu|}}{|\mu\beta|^2-3}$$

obtained by putting $\cos\Phi = \frac{2}{|\mu\beta|}$

The feedback must be negative; it can not be positive and meet this condition.

$$|\mu\beta| \geqq 2$$

Boundary H.—Stability with feedback equal to stability without feedback but opposite in sign

$$\left[\frac{\partial |A_F|}{|A_F|}\right]_{|\mu|} = \frac{\partial |\mu|}{|\mu|}$$

obtained by putting

$$\cos \Phi = \frac{2+|\mu\beta|^2}{3|\mu\beta|}$$

If

$$\cos\Phi = \frac{2+|\mu\beta|^2}{3|\mu\beta|},\ |1-\mu\beta|^2 = \frac{|\mu\beta|^2-1}{3}$$

The feedback must be positive; it can not be negative and meet this condition.

$$1 \leqq |\mu\beta| \leqq 2$$

Boundary F.—Stability is either improved or degraded according as the feedback is negative or positive by an amount corresponding to the decrease or increase respectively in gain due to feedback. Compared to the gain variation without feedback, the variation with feedback is of the same sign or opposite in sign according as the feedback is positive or negative

$$\left[\frac{\partial |A_F|}{|A_F|}\right]_{|\mu|} = \frac{\frac{\partial |\mu|}{|\mu|}}{|1-\mu\beta|^2}[1-|\mu\beta| \cos \Phi] = \frac{\frac{\partial |\mu|}{|\mu|}}{1-|\mu\beta|}$$

obtained by putting $\Phi=0°$. The feedback may be either positive or negative and meet this condition.

$$|\mu\beta| \geqq 0$$

As a further aid in analyzing the gain stability of feedback amplifiers, trace a path along one of the parametric contour lines of Fig. 2 and view the change in stability as Φ and $\mu\beta$ vary. For example, consider the 6 db negative feedback contour line; this means that the amplifier gain with feedback is 6 db less than without feedback. The amplification factor corresponding to 6 db is 2. If μ is increased to $\mu+\partial|\mu|$ and Φ remains constant,

$$100\frac{\partial |\mu|}{|\mu|}$$

is the percentage increase in output current without feedback. Starting at $\Phi=\pm 180°$ i. e. G, the per cent increase in output current with feedback is

$$\frac{1}{2}\left(100\frac{\partial |\mu|}{|\mu|}\right)$$

Moving along the -6 db contour toward E the stability is still further improved until at E the per cent increase in output current is

$$\frac{1}{4}\left(100\frac{\partial |\mu|}{|\mu|}\right)$$

Continuing along the -6 db line, the stability continues to improve until the contour line intersects C where the stability is perfect. As we continue further, the stability becomes poorer and the sign is reversed, (an increase in output without feedback corresponding to a decrease in output with feedback) until at F the change in output current becomes

$$-\frac{1}{2}\left(100\frac{\partial |\mu|}{|\mu|}\right)$$

As another example, assume that the above procedure is repeated except that the $+6$ db contour line is used instead of the -6 db line. Under these conditions the gain of the amplifier is 6 db more with feedback than without feedback. Without feedback, the percentage increase in current is

$$100\frac{\partial |\mu|}{|\mu|}$$

as before. Starting with $\Phi=0°$ and $|\mu\beta|=0.5$, i. e. at F, the percentage increase in current is

$$2\left(100\frac{\partial |\mu|}{|\mu|}\right)$$

or the stability is twice as poor with feedback as without feedback. Then as we move along the $+6$ db contour line in Fig. 2, the stability with feedback improves until as the contour line intersects B, the percentage increase has become

$$100\frac{\partial |\mu|}{|\mu|}$$

or is the same with feedback as without feedback. Then as we continue further, the stability becomes still better (is even better with feedback than without feedback although the feedback is positive) until the $+6$ contour line intersects C. Under these circumstances the stability is perfect. Beyond this point the stability changes sign (an increase in output without feedback corresponding to a decrease in output with feedback) until at H, the output current change in per cent is

$$-100\left(\frac{\partial |\mu|}{|\mu|}\right)$$

namely, the same as without feedback but opposite in sign. As we proceed further the stability continues to get worse (and remains reversed in sign) until at F the per cent change in output is

$$-2\left(100\frac{\partial |\mu|}{|\mu|}\right)$$

or the stability is twice as poor with feedback as without.

The first of the preceding examples illustrates the statement that with negative feedback, the gain stability is always improved and by an amount at least as great as would correspond to the gain sacrificed and generally more. This will be borne out in the section showing examples and results of experiments where it is practically always to be observed that the gain stability as measured is better than would be accounted for if the improvement corresponded only to the reduction in gain due to feedback.

Similarly, in the case of positive feedback, the gain stability is never degraded by more than what would correspond to the increase in gain due to feedback and under appropriate conditions, assuming the variations are not too great, is as good or much better than the gain stability without feedback.

Figs. 9, 10, and 11 have been prepared to still further indicate how the stability of amplification ratio and the gain stability are affected by the amount of feedback and the phase shift in once traversing the $\mu\beta$-path. Assuming small variations, Fig. 9 shows the effect of varying $|\mu|$ assuming $|\beta|$ and Φ fixed for various values of feedback and different values of Φ; the ordinates represent the number by which the stability of amplification ratio without feedback is multiplied to get the stability of amplification ratio with feedback.

Figs. 10 and 11 refer to finite variations in $|\mu|$ assuming $|\beta|$ and Φ fixed and that before $|\mu|$ is varied

$$|\mu\beta|=\frac{1}{\cos\Phi}$$

The figures are the loci of db-change in gain with feedback as a function of db-change in gain without feedback where the parameter is db-change in gain due to feedback before $|\mu|$ is varied. Fig. 11 is an enlargement of Fig. 10 in the neighborhood of the origin in order to indicate the gain stability of a feedback amplifier under favorable conditions when the effect of feedback is to give rise to considerable increase in gain.

Fig. 12 is a vector diagram of stability of amplification with and without feedback and illustrates the distinction between stability of amplification and amplification ratio or gain. It should be noted that to take advantage in this manner of improved stability over a band of frequencies may have a tendency to introduce a certain type of phase distortion although for many practical applications this could be tolerated. In the case of positive feedback the effect is further accentuated as may be judged by referring to Fig. 13 which is a vector diagram illustrating that, if with considerable positive feedback, the gain stability of the amplifier is materially improved; the improvement in stability will be accompanied by a substantial phase rotation or change in phase shift as viewed at the output of the amplifier.

Fortunately for the practical success and convenience of β-circuit equalization, for values of $\mu\beta >> \cos\Phi$, variations in the absolute magnitude of amplification ratio due to variations in β tend to become substantially independent of Φ. This is evident from Equation (n) which may be obtained in a manner similar to that used in deriving (l).

$$\left[\frac{\partial|A_F|}{|A_F|}\right]_{|\beta|}=\left|\frac{\mu\beta}{1-\mu\beta}\right|\left[\frac{\cos\Phi-|\mu\beta|}{|1-\mu\beta|}\right]\left|\frac{\partial\beta}{\beta}\right| \qquad (n)$$

This equation demonstrates that if $\cos\Phi=|\mu\beta|$ the magnitude of the output will not vary with small changes in $|\beta|$. This is boundary "B", Figs. 2, 3 and 4.

In like manner, the change in absolute value of amplification ratio expressed as

$$\frac{\partial|A_F|)}{|A_F|)_\Phi}$$

due to a small variation in Φ from Φ to $(\Phi+\partial\Phi)$ where $\partial\Phi$ is in radians is given by Equation (o)

$$\left.\frac{\partial|A_F|}{|A_F|}\right\}_\Phi=-\left|\frac{\mu\beta}{1-\mu\beta}\right|\frac{\sin\Phi}{|1-\mu\beta|}\partial\Phi \qquad (o)$$

This indicates that if $\mu\beta>>1$, the change in absolute value of amplification due to varying Φ is small irrespective of Φ. Furthermore, the change in amplitude of amplification due to a variation in Φ from Φ to $(\Phi+\partial\Phi)$ will be the same irrespective of whether $\partial\Phi$ is added to μ or β.

Equation (o) demonstrates that if Φ is equal to 0 or 180° then the magnitude of the output will not vary with small changes in Φ. This is boundary "F" and "G" Figs. 2, 3 and 4.

The stability of amplifier gain with feedback as affected by sufficiently small variations in $|\mu|$, $|\beta|$ and Φ occurring simultaneously is given by the sum of the variations due to each acting independently and alone and is given by differential Equation (p).

$$\left.\frac{\partial|A_F|}{|A_F|}\right\}_{|\mu|\,|\beta|\Phi}=\begin{cases}+\dfrac{\frac{\partial|\mu/}{|\mu|}[1-|\mu\beta|\cos\Phi]}{1-2|\mu\beta|\cos\phi+|\mu\beta|^2}\\[2ex]+\dfrac{|\mu\beta|(\cos\Phi-|\mu\beta|)}{1-2|\mu\beta|\cos\Phi+|\mu\beta|^2}\left|\dfrac{\partial|\beta|}{|\beta|}\right|\\[2ex]-\dfrac{|\mu\beta|\sin\Phi}{1-2|\mu\beta|\cos\Phi+|\mu\beta|^2}\partial\Phi \qquad (p)\end{cases}$$

In considering the effect of variations upon the absolute value of amplification ratio, if the variations are large it should be noted that as A_F is altered from A_F to $A_F+\Delta A_F$, it is

$$\frac{|A_F+\Delta A_F|-|A_F|}{|A_F|}$$

that corresponds to

$$\frac{\partial|A_F|}{|A_F|}$$

and not

$$\left|\frac{\Delta A_F}{A_F}\right|$$

Expressions will now be developed giving stability of gain with respect to finite variations in $|\mu|$ and $|\beta|$.

$G=20\log|\mu|$ = gain without feedback.

$G'=20(\log|\mu|+\log|\Delta\mu|)$ = gain without feedback after $|\mu|$ has changed by a factor $|\Delta\mu|$.

$G_F=20\left[\log|\mu|-\frac{1}{2}\log(1-2|\mu\beta|\cos\Phi+|\overline{\mu\beta}|^2)\right]$ = gain with feedback.

$G_F'=20\cdot\left[\log|\mu|+\log|\Delta\mu|-\frac{1}{2}\log(1-2|\mu\cdot\beta|\cos\Phi\cdot\Delta\mu+|\overline{\mu\cdot\Delta\mu\cdot\beta}|^2)\right]$ = gain with feedback after $|\mu|$ has changed by a factor $|\Delta\mu|$.

G_F-G_F' = change in gain in db due to change in $|\mu|$ with feedback.

$$G_F-G_F'=20\left[-\log|\Delta\mu|-\frac{1}{2}\log(1-2|\mu\beta|\cos\Phi+|\overline{\mu\beta}|^2)+\frac{1}{2}\log(1-2|\mu\beta|\cos\Phi\cdot|\Delta\mu|+|\overline{\mu\beta}|^2\cdot|\Delta\mu|)\right]$$

Let $A=2|\mu\beta|\cos\Phi$

$B=|\overline{\mu\beta}|^2$

$C=(1-A+B)=(1-2|\mu\beta|\cos\Phi+|\overline{\mu\beta}|^2)$

$$G_F-G_F'=10\cdot\log\left[\frac{1-A|\Delta\mu|+B|\overline{\Delta\mu}|^2}{C\cdot|\Delta\mu|^2}\right]$$

$$y=10\log y'$$

$$y'=\frac{1}{C}\cdot|\overline{\Delta\mu}|^{-2}-\frac{A}{C}|\overline{\Delta\mu}|^{-1}+\frac{B}{C}$$

Figs. 10 and 11 are the plots of this equation expressed in decibel changes.

III. Circuit Design Analysis of $\mu\beta$ Loop

As an aid to those skilled in the art, in designing circuits for practising the invention, the following discussion is given of criteria for determining whether an element is in the μ-circuit or in the β-circuit.

The reader who is interested in the general aspects of the invention rather than in actual circuit design may pass over this present section to section IV, and may disregard Figs. 14 to 20, inclusive, of the drawings.

III. a. Discussion of μ and β, μ-system and β-system, μ-circuit and β-circuit

It is to be noted that these terms result to some extent from the method of circuit analysis being used rather than from the generic configuration of the circuit.

The different properties already investigated of simple feedback systems have resulted always either in functions of two complex variables, μ and β, or in expressions which could be so interpreted. In what follows, μ-system and β-system will be used corresponding with μ and β as a convenient designation for the entire group of elements affecting μ or β respectively.

A test for determining to which complex variable an element of the system belongs is to assume that the element in question is varied and then to consider whether such a variation will alter μ or β (it can not affect both): the quantity affected signifies the complex variable to which the element varied belongs.

It is evident from what has gone before, that the functions μ and β are each associated with a different path or transmission system, called for convenience the μ-system and the β-system, respectively. It has also been disclosed that the two functions, μ and β, have very different properties.

Therefore, it becomes important from the standpoint of practical design to know whether a particular element is in the μ-system or in the β-system. This involves first the question of what portions of any given system are to be considered as concerned with either the μ-system or the β-system. Consider, by way of illustration, a five-stage amplifier with a feedback connection from the output of stage four to the input of stage two. It is obvious that in such a case the μ-system involved in the only feedback system that is present is not the whole five-stage amplifier but it includes only that portion lying between the points to which the feedback connection is made, that is, stages two, three and four. The μ-path or μ-system and the β-path or β-system together make up a closed loop. In the illustration, considering the feedback system, the input circuit is not the circuit attached to the input of stage one, but includes also stage one. Likewise, the output circuit of the feedback system includes stage five.

In stating that the μ and β-systems together make up a closed loop, there is no intention of limiting these to conductive paths nor to a closed loop in any narrow or restricted sense. In fact, any path of propagation is sufficient. It may include a radio link, or involve translation of energy from one form to another, as electrical to mechanical or radiant energy.

With this broad conception of the "closed loop" in mind, it is convenient (and not at all inconsistent) to represent, for purposes of discussion, the closed loop as a circular conductive circuit cut at two points, an input junction and an output junction. One portion of this loop transmitting in the direction from input to output junctions is the μ-system. The remaining portion whereby wave components are capable of returning from the output junction to the input junction is the β-system.

An element is in the μ-system or in the β-system according as it affects μ or β but not both.

Obviously, at the input and output junction points, three circuit branches are interconnected, μ-system, β-system and either the input or the output circuit as the case may be. It is at these junction points in particular that it may become difficult to determine in which system or path an individual element belongs.

It is convenient also to distinguish a μ-circuit and a β-circuit, analogous to μ-system and β-system. An element is in the μ-circuit or the β-circuit according as it, when varied, affects the output in the same way as μ or β affects the amplification.

It will be explained in detail hereinafter by reference to specific diagrams just how to determine in a typical case whether an element belongs in the μ-circuit or the β-circuit.

Thinking of a feedback system as a closed loop or transmission arrangement closed back upon itself, the propagation of a voltage in a single journey around such a closed path may be regarded as being represented by a single symbol, $T=\mu\beta$. A knowledge of T alone and lack of knowledge of μ and β would be adequate for testing the criterion for instability as given by Nyquist's rule, and the joining of the input and output circuits to the closed loop would be pertinent only insofar as T would be affected.

Viewing a simple feedback system as consisting of a single loop with input and output, as in Fig. 14 it is the connecting of the input and output circuits to the loop that in some manner has to do with dividing the transmission path into two parts, μ and β. With regard to this division of the transmission path, an input implies a source of excitation for the system and an output implies a voltage resulting from such excitation. In other words, μ and β have to do with a consideration of two voltages, one resulting from the cause or excitation and the other being the result or response of the system. Evidently T in

$$T=\mu\beta=\mu_x\beta_x$$

is independent of the excitation whereas the quantities μ_x and β_x vary, although their product is constant, depending upon what two voltages are viewed; the first voltage, however, always relating to the excitation and the second, to the response. Thus, it is the fact of a source of excitation of the system, the problem of where the excitation takes place, and the question as to where the result of this excitation is viewed as taking place that requires a consideration of the symbol T as being divided into two parts, μ and β.

If the plate circuit driving voltage in the equivalent circuit (for a linear system, the amplification

$$A_F=\frac{\mu}{1-\mu\beta}$$

could be interpreted as a measure of the properties of this voltage), were indicative of the performance of parts of the system and of the system as a whole from all points of view; then μ-circuit and β-circuit would be synonymous with μ and β or μ-system and β-system, and an element affecting μ would be in the μ-circuit and an element affecting β, in the β-circuit. This, however, is not always true. For example, in Fig. 1 and for a fixed input, the voltage across the output, L, might be regarded as a pertinent criterion of the performance of that amplifier. Assume, in Fig. 1, that R_0 is increased to

$$(R_0+\Delta R_0)$$

and μ is not altered. R_0 affects β[Equation (b)] and, hence

$$A_F=\frac{\mu}{1-\mu\beta}$$

is increased. It happens, as will be demonstrated subsequently, that the voltage across L is not materially changed as R_0 is increased (provided the increase is not too great and $\mu\beta>>1$) it being

$$\left(E_L+\frac{X}{1-\mu\beta}\right)$$

where E_L is this voltage before the change in R_0 was made and X is the amount by which this voltage across L would change due to the addition of ΔR_0 if the stabilizing effect of feedback were absent. Thus, although R_0 affects β; with respect to the voltage across L, it has the properties, as regards its effect on this voltage, that heretofore have been associated with μ in connection with its effect on A_F.

On the other hand, as regards certain higher order effects, such as modulation due to variations in R_0 with signal amplitude, R_0 behaves in part with respect to its effect on E_L exactly as β in its effect on A_F. (It can be shown, assuming $\mu\beta>>1$, that, as regards its effect on the output of the amplifier, the circuit may be arranged so that non-linear response in β is practically unaltered but reversed in sign and non-linear response in μ divided by $(1-\mu\beta)$.) Therefore, with respect to the linear contribution to the output current or voltage, R_0 is in the μ-circuit; with respect to modulation due to variations in R_0 with amplitude, R_0 in its effect on E_L partakes of the properties of both μ and β. In this case, however, it is to be noted that actually several output voltages are involved as a result of considerations which require separately viewing the non-linear response of the system to a fixed input: with respect to one of these voltages acting alone and for a specified direction of propagation around the $\mu\beta$-path, assuming as in Fig. 14 that the two directions of propagation will be considered separately, any element belongs to one circuit or system but not both.

Thus, for a specified excitation, an element is viewed as being in the μ-circuit or β-circuit with reference to its effect on a specified resulting voltage at some part of the system. Such an element will then be classified as belonging to the μ-circuit or β-circuit according to whether varying the element affects the voltage in question in a manner corresponding to the way an element of the μ-system or β-system respectively would affect a corresponding driving voltage in the last plate circuit of the equivalent circuit. In what follows, the terms μ-circuit and β-circuit will be described in connection with the output of the amplifier and it will be shown how to specify these circuits if it is known how to specify μ and β.

As a first step, it will be shown how to specify μ and β in a more general manner than has already been done in connection with Figs. 1, 5, 6, 7 and 8. Consider first only a single feedback path (systems involving multiple and repeated feedback arrangements will be considered hereinafter): Because at least a portion of the system or circuit comprises a transmission path which in some manner has been closed back upon itself in such a way that a voltage may be repeatedly propagated around this path, such a feedback system is represented, to begin with, as consisting of a closed loop, "O", with input and output as in Fig. 14. The input and output are in some manner to be joined to this closed path, "O", and $\mu\beta$ is to be regarded as the propagation encountered in making a single journey in a specified direction around the closed transmission path with input and output circuits joined.

The closed loop or path of one round trip journey is a single transmission circuit and it is the points of entry of the input or the points of exit of the output that divide it into two parts, μ and β or μ-system or β-system; and two circuits, μ-circuit and β-circuit. Connecting the input and output usually changes the propagation around the closed loop, "O"; an exception to this is the conjugate junctions of Fig. 5.

Considering μ and β or μ-system and β-system and a particular direction of propagation around the $\mu\beta$-path, the two points of entry of the input establish one boundary (Fig. 15) and the two points of exit of the output (Fig. 16), the other.

Furthermore, in accordance with the earlier presentation, μ and β could be determined if the points could be found at which the two voltages

$$V=V_1+V_2$$

and μV exist. The boundary of Fig. 15 or the two points, X_1, X_2, are the points across which the voltage is $V=V_1+V_2$.

To obtain a symbolic representation and an equivalent circuit, the output has been represented by a two terminal impedance, L, which is a complex quantity representing the impedance of the load at any frequency. On this basis there are plainly two wires which may be associated with the output.

One method for associating two wires with the input is to view the input connection in accordance with what is usually referred to as Thévenin's theorem whereby, as in Fig. 14, E is the open-circuit voltage and C, the internal impedance of the input. Irrespective of the method of excitation, it usually will be evident how, for the purpose of circuit analysis, the excitation may be simulated by a generator in an equivalent circuit.

Referring to Fig. 14, since the input is to connect to the closed path, there must be two points, designated x_1 and x_2, where this connection is made. As already mentioned, it is across these two points that the voltage is $V=V_1+V_2$.

There still remains the problem of determining the location of the internal generator, μV, of the equivalent circuit. Again referring to Fig. 14, inasmuch as it has been specified that the output or load is to be joined to the closed path "O", the two points where this connection is made have been designated y_1 and y_2.

In defining either of the pairs of points x_1, x_2 or y_1, y_2; it would have been more precise in each case to have said at least two points, for although it is plain that there would have to be two points rather than one point in order that the system be operative (if x_1 connects to x_2 or y_1 to y_2, either the input or output respectively will be short-circuited) there would be, for the case of either input or output, no reason why any wire or both wires could not have been connected to more than a single point in the feedback path and in this event, it will be shown (Figs. 15 and 16) how to determine in each case which one of the several points should be selected in preference to all others and designated in the manner described as a point of entry or exit. If either x_1 and y_1 or x_2 and y_2 should coincide, separately or simultaneously, the point in question receives both designations.

The direction of propagation around the $\mu\beta$-path has been specified because both directions should be considered unless, because of a tube or other practically unilateral device, the loop transmission is restricted to a single direction. If $\mu\beta$ has a finite value in each direction, one method for taking into account the two directions would be to consider one direction at a time and then apply the principle of superposition. Fig. 15 indicates, depending upon the direction, how to locate x_1 and x_2 in case the input does not connect to the loop path in the simplest possible manner. It is to be noted that in Fig. 14, the μ-system is bounded at a or b according as the upper or lower portion of the loop path is the μ-circuit which will be according as to whether the transmission around the closed loop is clockwise or counter-clockwise; this is because it is assumed that the transmission sought is from input to output and the formulae previously presented have been derived by assuming that upon a single journey around the $\mu\beta$-path, starting at (x_1, x_2), a voltage is modified first by μ and then β.

Referring to Fig. 15, to locate (x_1, x_2) when an input wire joins the loop path at several points, the rule is to move with the transmission and the last point passed is the point in question. Fig. 16 illustrates how to locate y_1 and y_2; the rule is similar except to move against the transmission.

It is to be noted that for a particular direction of propagation around the $\mu\beta$-path; elements of the loop path, input and output (Fig. 14) are never common to both μ and β or μ-system and β-system: they may belong to one or the other but not to both. On the other hand, elements of the β-system may be common to the β-system and to the path between input and loop path or loop path and output. Acting in this manner (Figs. 15 and 16) such elements introduce insertion loss or gain between the input equivalent driving voltage, E, and the voltage, V, across the points of entry (x_1, x_2); or between the equivalent driving voltage in the plate circuit of the last tube, μV, at the points (y_1, y_2) and the voltage across the output, L. The transmission changes thus effected are not altered by the feedback properties of the system.

It is to be noted further that the elements involved in producing these insertion effects are bounded by boundaries (x_1', x_2') and (x_1, x_2) for the input (Fig. 15), and (y_1, y_2) and (y_1', y_2') for the output.

The determination of the μ-circuit and β-circuit is somewhat involved but will be mentioned because of its importance. In this connection, reference will be made to Fig. 17, which shows an equivalent circuit of a simple feedback amplifier applying the principles of Figs. 14, 15 and 16. The μ-circuit consists of the elements in the μ-system plus the elements in the β-system that affect the load voltage and the voltage fed back in exactly the same way. The β-circuit consists of elements in the β-system that are not removed as part of the μ-circuit. It will be noted that parts of the μ-system are never a part of any other transmission path and do not appear at all in the equivalent circuit. With a fixed input or excitation, the equivalent circuit is the $\mu\beta$-path, which includes input and output of Fig. 14, broken by the removal of the μ-system. The addition of a generator μV simulates the μ-system and consequently the effect of feedback. As a result, the μ-circuit is composed of the μ-system plus certain elements of the β-system.

With regard to the output, the only elements of the β-system that could possibly be eligible for removal would be those included between boundaries (y_1, y_2) and (y_1', y_2') of Fig. 16. Except in special cases, it is necessary to examine each eligible element. To be in the μ-circuit, the criterion to be satisfied is as follows: vary the element under consideration and view the effect of such a change upon β as $(\beta+\Delta\beta)$ and upon the transmission from μV to E_L, the voltage across L, as $E_L+\Delta E_L$; if

$$\frac{\Delta E_L}{E_L}=\frac{\Delta\beta}{\beta}$$

the effect of the variation upon β and upon the propagation from μV to L will be identical, and the element in question is in the μ-circuit.

If $\mu'\mu V=E_L$, then

$$x=\frac{\mu'\mu}{1-\mu\beta}$$

is the voltage amplification from the grid of the first tube to L and when the element in question varies from Z to $(Z+\Delta Z)$ then

$$x+\Delta x=\frac{(\mu'+\Delta\mu')\mu}{1-\mu(\beta+\Delta\beta)}$$

where

$$\frac{\Delta E_L}{E_L}=\frac{\Delta\mu'}{\mu'}=\frac{\Delta\beta}{\beta}$$

if the rule is to be satisfied. If the changes are small,

$$\left.\frac{\partial x}{x}\right)_Z=\frac{\frac{\partial\mu'}{\mu'}}{1-\mu\beta}$$

which is the same as if, instead of varying the element in question, μ had varied from μ to $(\mu+\partial\mu)$ where $\partial\mu$ satisfies the relationship

$$\frac{\partial\mu}{\mu}=\frac{\partial\mu'}{\mu'}$$

as may be seen from Equation (j). If the changes are large,

$$\left.\frac{\Delta x}{x}\right)_Z=\frac{\frac{\Delta\mu'}{\mu'}}{1-\mu(\beta+\Delta\beta)}$$

and now if

$$\frac{\Delta\mu'}{\mu'}$$

is replaced by

$$\frac{\Delta\mu}{\mu}$$

we have

$$\frac{\Delta x}{x}=\frac{\frac{\Delta\mu}{\mu}}{1-(\mu+\Delta\mu)\beta}$$

which is the same as Equation (g). Therefore, properties of the μ-circuit are exactly the same as what have been termed properties of the μ-system.

In Fig. 17 R_0 is in the β-system but satisfies the above criterion and, hence, is also in the μ-circuit. The importance of this is that as a result of being in the μ-circuit, the effect of variations in R_0 upon amplification and non-linear response may be stabilized for variations in R_0 as effectively as if R_0 were in the μ-system.

With regard to the input, the situation is similar to that for the output. The elements eligible for consideration are those between boundaries (x_1, x_2) and (x_1', x_2') of Fig. 15. The rule is that a variation in one of these elements is liable to affect both β and V_1. If, the result of a change in the element in question is to make

$$\frac{\Delta V_1}{V_1}=\frac{\Delta\beta}{\beta}$$

then the element belongs to the μ-circuit. Accordingly, as one of these elements belonging to the μ-circuit is varied, the effect of its variation on the grid voltage is altered by feedback in the manner that μ is altered by feedback, namely, by dividing by $(1-\mu\beta)$.

All elements of the β-system not in the μ-circuit are eligible for consideration as to whether or not they are in the β-circuit. The requirement to be satisfied is that if the system be excited (input applied) and the element in question varied, the effect of its variation upon the voltage being viewed shall be nil without feedback. The voltage viewed without feedback is to be interpreted as the limit the voltage at the same place and with feedback would approach if $\beta \doteq 0$ and simultaneously there would be no change anywhere in the passive impedances of the complete system. To satisfy this condition, the element in question apparently would either have to be conjugate to the voltage being viewed (output) or have to be shielded from it by a tube or other unilateral device. In all other cases, the extent to which an element of the β-system partakes of the properties of the β-circuit is a matter of degree.

For example, in Fig. 1, if

$$f<<\frac{CG_0}{C+G_0}+\frac{R_0L}{R_0+L}$$

it approaches meeting the requirement for being in the μ-circuit; on the other hand, if

$$f>>\frac{CG_0}{C+G_0}+\frac{R_0L}{R_0+L}$$

it approaches meeting the requirement to be satisfied for belonging to the β-circuit. It is to be noted that even for a fixed configuration of the β-system, the nature and value of all its elements may have to do with the question as to which circuit, μ-circuit or β-circuit, if either, an element belongs.

Figs. 18, 19 and 20 are presented to illustrate further the application of these concepts regarding μ, β, μ-system, β-system, μ-circuit, and β-circuit to the drawing of equivalent circuits not closed back upon themselves and yet simulating feedback properties.

Fig. 18 applies these methods of analysis to an amplifier that feeds back through bridges as in Fig. 5 described above, by way of example; Fig. 19 applies these methods of analysis to a case of a simple voltage-voltage feedback, as another example; and Fig. 20 applies these methods of analysis used in Figs. 14, 15, 16 and 17 to the case of a simple common impedance feedback, as a further example.

In the case of Fig. 19, all elements C, G, R, L, and f affect β [see Equation (b)] and do not change μ.

Despite the fact that μ is stabilized and β is not (as shown above in discussion of Figs. 6 to 8): for a fixed input, E, and $\mu\beta>>1$, variations in G, R_0 and L produce no material change in the voltage across L. However, a percentage change in f, assuming f is sufficiently large, produces a substantially equal percentage change in the voltage across L (if f is increased the voltage is increased and vice-versa).

In the case of Fig. 20 the load impedance L does not change the voltage fed back in the same ratio that it changes the voltage across the load. Therefore, the load voltage is not stabilized by feedback against variations in the load impedance to the same extent that it is in Fig. 19. In Fig. 20, negative feedback stabilizes load current against variations of load impedance; whereas, in Fig. 19, negative feedback stabilizes load voltage against those variations.

It is to be noted that μ is not restricted to values greater than β as in the examples cited. In Fig. 5, if the amplifier is turned around and the amplifier and network circuits interchanged, the over-all transmission is a loss instead of a gain. In this case μ is a loss and β a gain and the entire procedure suggests a way of building a stable artificial line useful for special purposes.

Moreover, both μ and β can be losses; and both μ and β can be gains.

IV. Modulation or Distortion

The complexities surrounding the evaluation of the flow of current when the response of the system is not linear are known and for the case of an amplifier without feedback, the subject has been treated elsewhere. ("Theory of Three-Electrode Vacuum Tube" by John R. Carson, I. R. E. Proc., vol. 7, April 1919 pp. 187–200; "The Equivalent Circuit of the Vacuum Tube Modulator" by John R. Carson, I. R. E. Proc., vol. 9, June 1921 pp. 243–249; "Notes on Theory of Modulation" by John R. Carson, I. R. E. Proc. vol. 10, February 1922 pp. 57–64; "Operation of Thermionic Vacuum Tube Circuits" by F. B. Llewellyn, The Bell System Technical Journal, vol. 5, No. 3, July 1926, pp. 433–462.) Herein, the objective will be to discuss not the non-linear response without feedback but how the non-linear response without feedback is modified by the feedback properties of the system.

Fig. 21 shows a closed loop in which the propagation in one complete journey around the loop changes a voltage by the factor

$$\mu\beta,\text{ i. e. } |\mu\beta|\underline{|\Phi}$$

For convenience, reference is now made to this figure to demonstrate that to a first approximation, if D is the distortion (harmonics or modulations) without feedback,

$$\frac{D}{1-\mu\beta}$$

is the distortion with feedback. Explanation is given hereinafter of the necessary conditions surrounding the arrangement of the circuit in order that the operation approximate this result. This demonstration is applicable not only to the system of Fig. 21 but to feedback systems in general.

Assuming the above conditions are satisfied, an undesired distortion voltage in the μ-circuit will be replaced by an equivalent generator in one of the shunt impedances. Let this generator be "D" when the feedback path (or loop) is opened without disturbing the impedance relations. With feedback (or when the loop is closed) it is possible to visualize two generators in series with this impedance. One is "D" the other is that portion of the sum of these two that traverses the loop and is altered by the factor $\mu\beta$. Call this second generator $\mu\beta D'$ where

$$D'=D+\mu\beta D'$$

then

$$D'=D\left(\frac{1}{1-\mu\beta}\right)$$

which demonstrates that under the conditions assumed any distortion voltage without feedback will be changed by the factor

$$\left(\frac{1}{1-\mu\beta}\right)$$

with feedback.

Thus, under these circumstances it is to be noted that if the system is operated with a constant output of the fundamental then the magnitude of the fundamental at any point in the μ-circuit is the same with or without feedback. But the distortion introduced by this fundamental is changed by the quantity

$$\left(\frac{1}{1-\mu\beta}\right)$$

Continuing the preceding analysis, consideration will now be given to ascertaining distortion levels at various parts of a feedback system. In this connection reference will be made to the amplifier system of Fig. 22 as illustrative, and harmonic or distortion levels at various parts of the amplifier with and without feedback will be found, assuming that the first two stages do not contribute any distortion and that the fundamental output is constant.

In connection with this figure, the μ's as shown have been used to have special meanings, and different from the general definition and meaning employed for μ elsewhere in this specification. These special meanings are defined as follows:—

(generator at 1)$\cdot\mu_1$=(generator at 2)
(generator at 1)$\cdot\mu_2$=(generator at 3)
(generator at 1)$\cdot\mu$ =(generator at 4)

These relations follow from the usual assumption that a fictitious generator in the plate circuit has a voltage equal to that of a fictitious generator in the grid circuit multiplied by the amplification factor involved.

For simplicity assume only one distortion introduced, and that at point 4. Under this condition to compare the distortion with feedback at various parts replace the distortion at that place by an equivalent generator and compare these equivalent generators. This gives:

Distortion generator at point 4
Without feedback$=D$

Distortion generator at point 4
With feedback$=D/1-\mu\beta$

Distortion generator at point 1
Without feedback$=0$

Distortion generator at point 1
With feedback$=\beta\cdot D/1-\mu\beta$

Distortion generator at point 2
Without feedback$=0$

Distortion generator at point 2
With feedback$=\dfrac{\mu_1\beta D}{1-\mu\beta}$

Distortion generator at point 3
Without feedback$=0$

Distortion generator at point 3
With feedback$=\dfrac{\mu_2\beta D}{1-\mu\beta}$

Distortion generator at point 4
With feedback$=\dfrac{\mu\beta D}{1-\mu\beta}+D=D/1-\mu\beta$

That is, assuming as the only distortion a distortion D in the plate circuit of the final stage, the distortion at each point indicated in the circuit is found as a certain proportion of D depending on the values of the μ and β involved in the closed loop for the given point when there is a feedback.

Regarding modification of non-linear response by feedback, one of the more important examples to be considered is arrangements of negative feedback whereby the modulation is improved (i. e. the effect of unwanted modulation is reduced) as the gain is reduced. In such cases the output is assumed fixed and the analysis is directed toward a comparison of the difference in modulation with and without feedback.

To begin with, it will be suggestive, assuming no non-linearity, to view the physical aspects of feedback with regard to its effect upon the grid voltage. Without feedback the grid voltage has been designated V_1 and with feedback

$$V_1+V_2=V_1+\frac{\mu\beta}{1-\mu\beta}V_1$$

For values of $\mu\beta$ large in comparison with unity, the feedback voltage, V_2, is practically equal and opposite to V_1 (the value without feedback) irrespective of the phase shift around the $\mu\beta$-path. Negative feedback as obtained in this manner is dependent upon $\mu\beta$ being sufficiently large rather than upon the phase shift around the amplifier and network circuits. However, as

$$\mu\beta=|\mu\beta|\underline{|\Phi}$$

is increased without limit,

$$\frac{\mu\beta}{1-\mu\beta}$$

approaches −1 although in absolute magnitude it may be a little less or a little more or for two values of Φ equal to −1. The sum of these two nearly equal and opposite voltages is

$$\left(1+\frac{\mu\beta}{1-\mu\beta}\right)V_1$$

and thus always equals

$$\frac{V_1}{1-\mu\beta}$$

In other words, the effect of considerable negative feedback is to reduce the gain by feeding back a voltage almost equal to and out of phase with the grid voltage without feedback. This is further illustrated by Fig. 23, which is a vector diagram of the relations between the applied voltage (unit vector), the voltage feedback

$$\left(\frac{\mu\beta}{1-\mu\beta}\right)$$

and the resulting voltage

$$\left(\frac{1}{1-\mu\beta}\right)$$

for positive and negative feedback. In Fig. 23 it is to be observed that although for large values of $\mu\beta$ the argument of

$$\frac{\mu\beta}{1-\mu\beta}$$

always approaches 180°,

$$\frac{1}{1-\mu\beta}$$

which is the resultant or final voltage on the grid with feedback, may assume any angle from 0° to 360° depending upon Φ and $|\mu\beta|$.

Another point to be noted is that if

$$\mu(V_1)_0=\frac{\mu V_1}{1-\mu\beta}$$

meaning that the outputs are alike with or without feedback, the voltages everywhere within the

μ-system with feedback are the same as without feedback.

Now, it will be assumed for the moment that the non-linear response is due to μ and that β is linear. The method used will be one of successive approximation. As the first approximation μ is assumed linear: the grid voltage without feedback is $(V_1)_0$ and the plate voltage $\mu(V_1)_0$. With feedback and a change in input from $(V_1)_0$ to V_1, the grid and plate voltages are

$$\left(V_1+\frac{\mu\beta}{1-\mu\beta}V_1\right)$$

and

$$\mu\left(V_1+\frac{\mu\beta}{1-\mu\beta}V_1\right)$$

where

$$\mu(V_1)_0=\mu\frac{V_1}{1-\mu\beta}$$

As a second approximation account is taken of the non-linear response without feedback when the grid voltage is $(V_1)_0$. This is represented in the analysis as μD_1 which acting in series with R_0 in the equivalent circuit is to produce the same distortion (amplitude and phase) in the load as is actually obtained from the operation of the amplifier without feedback when the grid voltage is $(V_1)_0$. As a second approximation term with feedback which is to be added to the first approximation with feedback, the effect of adding a generator μD_1 in series with R_0 is evaluated. This can readily be done; for, as will now be shown, feedback changes the effect of a generator voltage in the plate circuit of an amplifier in the same way that it changes the amplification of the system, namely, by dividing by $(1-\mu\beta)$.

Considering, for example, the circuit of Fig. 24, which is a feedback amplifier circuit similar to that of Fig. 5 with the β-circuit network omitted, and which shows a fictitious plate generator μD_1 representing the non-linear response to a first approximation, X designates the resulting voltage that feedback causes on the grid; and the values of X and of the resultant plate voltage $(\mu D_1+\mu X)$ can be obtained as follows:

$$X=\beta(\mu D_1+\mu X)=\mu\beta D_1+\mu\beta X$$

$$X(1-\mu\beta)=\mu\beta D_1 \text{ or } X=\frac{\mu\beta}{1-\mu\beta}D_1$$

$$\mu D_1+\mu X=\mu D_1\left(1+\frac{\mu\beta}{1-\mu\beta}\right)=\frac{\mu D_1}{1-\mu\beta}$$

Thus, the above-mentioned evaluation of the effect of adding a generator μD_1 in series with R_0 is

$$\frac{\mu D_1}{1-\mu\beta}$$

indicating that the approximate effect of feedback is to modify this generator μD_1 in the same way that it modifies the gain. Accordingly, the second approximation term with feedback which is referred to above as to be added to the first approximation with feedback, is

$$\frac{\mu D_1}{1-\mu\beta}$$

and this has been indicated in the following table of successive approximations. It is to be noted that by a method of successive approximations the results with feedback may be determined if it is known how to compute the results without feedback. It is to be noted that in practical applications the approximation converges with extreme rapidity.

TABLE

Showing how the modulation with feedback is related to the modulation without feedback and may be evaluated by a method of successive approximations

Approximation	Grid voltage		Plate generator	
	Without	With	Without	With
1st	V	V	μV	μV
2nd	$V+0$	$V+\frac{\mu\beta}{1-\mu\beta}D_1$	$\mu V+\mu D_1$	$\mu V+\frac{\mu D_1}{1-\mu\beta}$
3rd	$V+\frac{\mu\beta}{1-\mu\beta}D_1$	$V+\frac{\mu\beta}{1-\mu\beta}D_2$	$\mu V+\mu D_2-\frac{\mu\beta}{1-\mu\beta}\mu D_1$	$\mu V+\frac{\mu D_2}{1-\mu\beta}$
4th	$V+\frac{\mu\beta}{1-\mu\beta}D_2$	$V+\frac{\mu\beta}{1-\mu\beta}D_3$	$\mu V+\mu D_3-\frac{\mu\beta}{1-\mu\beta}\mu D_2$	$\mu V+\frac{\mu D_3}{1-\mu\beta}$
nth	$V+\frac{\mu\beta}{1-\mu\beta}D_{n-2}$	$V+\frac{\mu\beta}{1-\mu\beta}D_{n-1}$	$\mu V+\mu D_{n-1}-\frac{\mu\beta}{1-\mu\beta}\mu D_{n-2}$	$\mu V+\frac{\mu D_{n-1}}{1-\mu\beta}$

This table is offered as an aid in presenting a physical concept of the final result of the successive approximation and to show the physics of why feedback improves the modulation. It is to be noted that feeding back fundamental is not essential to and plays no part in the process, it being the propagation of the distortion components around the $\mu\beta$-path that leads to a reduction in distortion in the output of the amplifier. By a series of repetitions of the feedback process, it is possible to construct an amplifier so that the effect of feedback is to change the gain of fundamental but not affect the distortion; or to change the distortion but not the fundamental; or to affect both fundamental and distortion either by like or unlike amounts; or to use combinations of any or all the three possibilities: for example, to increase the gain by a large amount and not change the distortion and then to reduce the distortion and not change the gain.

Although the method of successive approximation yields the final result, it is advantageous to point out why under certain conditions (for the case of a single $\mu\beta$-path and $\mu\beta>>1$) the second and third harmonics, for example, may not be improved by the same amount as the gain is reduced. In the first place, new frequency voltages are across all the grids with feedback that without feedback might even be entirely absent if, for example, the production of harmonics were confined to the plate circuit of the last tube. Secondly, if the modulation is not confined to output of the last tube, these voltages may be distributed differently across the various parts of the μ-system with feedback as compared to the case without feedback. Thirdly, in general the modulation at one frequency depends upon the flow of current at another frequency. As an illustration of this, the third harmonic might be improved by the flow of second harmonic and then if feedback practically suppresses the second harmonic, the third harmonic is made worse and while the effect of feedback may be to improve the third harmonic, the third harmonic it improves has already been made worse. Finally, R_0 is in the β-circuit and if the variations in its value with amplitude are too excessive there is a limit to the improvement due to negative feedback around a single $\mu\beta$-path because the modulation in the β-circuit is not improved.

It is pointed out in passing, that in a feedback amplifier circuit using an input bridge (as in Figs. 5 or 24, for example), for large values of negative feedback, the ratio of distortion or harmonics to signal when measured across the feedback diagonal of the input bridge (i. e., across the arms K'G and K'G_0 of Fig. 5 in series) is approximately the same as the ratio of these components measured in the output circuit without feedback. This suggests a way of checking the condition of the tubes (as regards their harmonic production without feedback) without opening the feedback path, which is undesirable, or taking the amplifier out of service. This may be done by connecting a suitable measuring set across the feedback diagonal of the input bridge and determining the ratio of harmonics to signal.

V. Impedance as Affected by Feedback

V. a. Generalized impedance relationship in a feedback amplifier having a single $\mu\beta$ path

In considering the impedance offered by any two points in such a system, it will be assumed that an applied voltage or generator in series with an impedance is connected to the points involved. The impedance across these points at any frequency is herewith defined as the ratio of the voltage across the points to the current entering, the latter being the resultant current in the applied generator impedance.

Representing the voltage and current at any frequency by complex quantities, the complex measure of these vector quantities, their ratio will be a complex quantity which, in general, is a function of frequency and leads to a value of impedance that is the same as the impedance that would be measured by an impedance bridge. Therefore, the impedance as measured by an impedance bridge will depend upon the impedance of this bridge only if the impedance in question as defined depends upon the generator impedance. If it is assumed that the system is linear, it is to be expected that there would be no cases in which the measured impedance would depend upon the impedance of the measuring circuit, provided the system is stable when facing the measuring impedance, namely, the impedance of the generator. The impedance to be determined thus appears to be independent of the impedance of the generator or measuring circuit with the single restriction that the generator impedance does not either so alter $\mu\beta$ as to cause this altered value of $\mu\beta$ at any frequency to equal $(1+j0)$, or, more generally, that it does not lead to a new propagation characteristic that fails to comply with Nyquist's rule for freedom from instability.

The impedance with feedback may differ from the impedance without feedback because the effect of feedback is to alter the voltage to current ratio already defined as impedance unless the feedback is conjugate to the impedance in question. Referring to Fig. 5, examples of conjugate arrangements would be the input and output impedance with C and L respectively face.

In considering impedance, it has been assumed that the effects of non-linearity may be neglected. If the system is not sufficiently linear to justify neglecting these effects, then the relationships become more complicated and the impedance at any one frequency depends upon the impedance of elements of the system at other frequencies and, in addition, upon the amplitude and character of the applied generator voltage as well as the impedance-frequency characteristic of the generator or measuring circuit.

Figs. 25, 26, 27, 28 and 29 illustrate, in particular instances, the effect of feedback upon impedance.

In these figures, what is meant by the impedance without feedback is the value which the impedance with feedback approaches as μ is caused to approach zero; or is the value of the impedance in question if μ could in some manner be reduced to zero and, at the same time, the remaining properties of the $\mu\beta$-path not be altered.

Referring to Fig. 25, if the complex transmission ratio for a voltage transmitted once around the loop is $\mu\beta$, then with the $\mu\beta$-path closed through, the impedance seen across the interstage between grid and cathodes (ground) is

$$Z=\frac{Z_0}{1-\mu\beta}$$

where Z is the impedance with feedback and Z_0 is the impedance without feedback or when $\beta=0$.

The impedance will not change with different values of bridge impedance and the value of $\mu\beta$ that must be used is with no bridge across the circuit. The only restriction upon the value of impedance looking back into the bridge used for measuring is that it must not produce instability in the amplifier and feedback circuits at any frequency.

Referring to Fig. 26, if a feedback amplifier circuit is cut in a series arm as shown and the impedance measured with and without feedback, we get the relation

$$Z=Z_0(1-\mu\beta)$$

where Z_0 is the impedance without feedback, Z the impedance with feedback, and $\mu\beta$ the complex transmission ratio once around the loop before the cut was made.

The value of Z above will not depend upon the impedance of the bridge used to measure it. If the system is made up entirely of unbalanced π type structures this relation will hold for a cut in any series arm.

Referring to Fig. 27, if the shunt arm Z_L is cut and the impedance measured with and without feedback, we get the relation

$$Z=\frac{R_0}{1-\mu\beta\frac{R_0+Z_L}{Z_L}}+Z_L$$

and

$$Z_0=R_0+Z_L$$

and

$$Z=Z_0\left[\frac{Z_L}{Z_L-\frac{\mu\beta}{1-\mu\beta}R_0}\right]$$

where Z_0 is the impedance without feedback, Z is the impedance with feedback, and $\mu\beta$ is the complex transmission ratio around the loop before the shunt arm was cut.

It will be noticed that

$$\mu\beta\left(\frac{R_0+Z_L}{Z_L}\right)$$

is the value which the propagation around the feedback path would have if Z_L were open. Then if the shunt arm is opened we see R_0 modified by the feedback in the ratio

$$\frac{1}{1-\mu\beta\left(\frac{R_0+Z_L}{R_0}\right)}$$

and with Z_L in series with it. This impedance is not affected by the impedance of the bridge used to measure it.

Referring to Fig. 28, if the impedance seen across the series arm Z_S is measured we get the relations

$$Z_0=\frac{(R_0+Z_L)Z_S}{R_0+Z_L+Z_S}$$

$$Z=Z_0\left(\frac{1-\mu\beta\frac{Z_S}{Z_0}}{1-\mu\beta}\right)$$

where Z_0 is the impedance without feedback, Z the impedance with feedback and $\mu\beta$ the propagation around the $\mu\beta$-path.

The value of Z is independent of the bridge impedance and the above relation may be applied to any part of a system made up of unbalanced π type sections.

The value of Z is the same as would be obtained if we viewed the value of (R_0+Z_L) modified by multiplying by $(1-\mu\beta)$ as in Fig. 26 and as would be the case if Z_S were zero and then Z_S considered as being in parallel with that impedance.

Referring to Fig. 29, if the impedance is measured across a part of a shunt arm we get the relation

$$Z=Z_0\left[1+\frac{Z_2R_0}{(Z_1+Z_2)(R_0+Z_1)}\left(\frac{\mu\beta}{1-\mu\beta}\right)\right]$$

where Z_0 is the impedance without feedback, Z is the impedance with feedback, and $\mu\beta$ is the propagation around the $\mu\beta$-path.

It should be noted that the generalized definition of the impedance without feedback is considered to be the limit which the impedance with feedback approaches as $\mu\doteq 0$ and no other property of the $\mu\beta$-path is altered, whereas the amplification without feedback is obtained by considering the limit which the amplification with feedback approaches as $\beta\doteq 0$ and no other property of the $\mu\beta$-path is altered. (The fact that in Fig. 26, for example, the impedance without feedback may be obtained by making $\beta=0$ is to be interpreted as merely another way of approaching the impedance limit mentioned which is valid in this and the other examples but not generally.)

The fact that for certain conveniently accessible parts of the system, the relationship between the impedance with feedback and the impedance without feedback is related to $\mu\beta$ in a simple manner and is independent of the impedance measuring circuit, suggests a practical way of measuring $\mu\beta$, both modulus and angle, without opening the $\mu\beta$-path and under any desired condition of excitation. Such an impedance would be measured with and without feedback and $\mu\beta$ thus determined.

A first conclusion can be drawn from a consideration of examples such as those given in Figs. 25 to 29, namely, that if a conductor is cut and a measuring circuit is connected to the two terminals formed by the cut ends, the impedance of the circuit across these two terminals approaches zero as $\mu\beta$ approaches unity. This is believed to be a general rule for all points which are actually included in the $\mu\beta$ path. This means that for any finite voltage, the response would be infinite which is the familiar case of instability.

A second conclusion can be drawn, namely, that the impedance measured by bridging across two points separated by a finite impedance becomes infinite if the value $\mu\beta$ becomes unity. It is thought that this also is true for any two points which are actually in the $\mu\beta$ path. This conclusion is consonant with the first conclusion, and the two are analogous to the cases of series and parallel resonance.

V. b. Equations for the input and output impedances corresponding to the configurations of Figs. 30 to 34

While the generalized relationships discussed under the preceding heading may be used as an aid in deriving specific equations, for the case of the circuit configurations of Figs. 30 to 34, specific equations have been derived without recourse to the generalized relationships in order to show the general method of derivation. The configuration of Figs. 30 to 32 is termed voltage-voltage feedback and will be designated configuration A, and that of Figs. 33 and 34 is termed current-current feedback and will be designated configuration B. These expressions are preferred, although the term "parallel-parallel" could be used for configuration A and the term "series-series" for configuration B. In either case, the significance is thought obvious, namely, that the feedback circuit is connected to derive a voltage from across the output (or from a current flow in the output) and to apply a voltage across the input (or set up current flow in the input). The impedance properties of these particular circuits are also discussed in detail and are used to emphasize the difference between amplification with and without feedback and impedance with and without feedback.

Figs. 30 to 34 show vacuum tube amplifier circuits in which *g* is the grid of the first tube and *p* is the plate of the last tube. Figs. 31A, 32A and 33A show equivalent or simplified forms of the circuits (or of portions of the circuits) of Figs. 31, 32 and 33, respectively. All symbols used in these figures and in the derivations of equations in connection therewith are complex quantities presented as complex measures of certain vector quantities, impedances or generalized voltage-coefficients. Z_0 is used to designate impedance without feedback, and Z is used to designate impedance with feedback.

Referring to Fig. 30,

$$Z_0=\frac{G_0\left(f+\frac{R_0L}{R_0+L}\right)}{G_0+f+\frac{R_0L}{R_0+L}}\text{ input impedance without feedback}$$

$$V=V_1+V_2=V_1+\frac{\mu\beta}{1-\mu\beta}V_1=\frac{V_1}{1-\mu\beta}$$

$$I=\frac{E}{C+Z_0}-\frac{\mu\beta}{1-\mu\beta}\frac{V_1}{C}=I_1-\frac{\mu\beta}{1-\mu\beta}\left(\frac{Z_0}{C}\right)I_1$$

$$Z=\frac{\frac{V_1}{1-\mu\beta}}{I_1-\frac{\mu\beta}{1-\mu\beta}\frac{Z_0}{C}I_1}=\frac{Z_0}{1-\mu\beta}\left[\frac{1}{1-\frac{\mu\beta}{1-\mu\beta}\frac{Z_0}{C}}\right]$$

$$Z=\frac{Z_0}{1-\mu\beta\frac{C+Z_0}{C}}=\frac{Z_0}{1-(\mu\beta)_z}\text{ input impedance with feedback}$$

The symbol $(\mu\beta)_Z$ is introduced here and will be used with reference to impedance in the same general way that the symbol $\mu\beta$ is used with respect to amplification. The value of the quantity $(\mu\beta)_Z$ will evidently depend upon the location of the impedance considered.

The amplifier circuit of Fig. 1 corresponds to configuration A and, as shown above, the impedance the connecting cable or circuit C faces with feedback is related to the impedance faced without feedback by the equation,

$$Z=\frac{Z_0}{1-\mu\beta\left(\frac{C+Z_0}{C}\right)}\qquad(q)$$

where

$$Z_0=\frac{G_0\left(f+\frac{R_0L}{R_0+L}\right)}{G_0+f+\frac{R_0L}{R_0+L}}\qquad(r)$$

and

Z_0=the impedance without feedback.
Z =the impedance with feedback.

To show that (1) is independent of C,

$$\beta=\left(\frac{L}{R_0+L}\right)\frac{\frac{CG_0}{C+G_0}}{\frac{R_0L}{R_0+L}+f+\frac{CG_0}{C+G_0}}\qquad(b)$$

which may be rewritten,

$$\beta=\left(\frac{L}{R_0+L}\right)\frac{G_0}{G_0+f+\frac{R_0L}{R_0+L}}\left(\frac{C}{Z_0+C}\right)\qquad(s)$$

From (q) and (s), it is plain that Z is independent of C.

It is also evident that Z does not approach

$$\frac{Z_0}{1-\mu\beta}$$

except as

$$\frac{C+Z_0}{C}\doteq1.$$

This situation is similar to the discussion above in connection with Figs. 25, 26, 27, 28 and 29, inasmuch as the transmission around the feedback path is affected by the impedance of the exciting generator. In this case, the impedance of the generator has been noted C. However, Z (the impedance with feedback) equals Z_0 (the impedance without feedback) divided by a quantity $(1-\mu\beta)$. The $\mu\beta$ to be used in this case is the $\mu\beta$ of the system when the exciting generator together with its impedance C is short-circuited. Attaching the bridge or excitation does not change the impedance measured by the bridge or the impedance presented to the exciting generator.

It is to be observed that the $\mu\beta$ involved in the above expression

$$\frac{Z_0}{1-\mu\beta}$$

is the $\mu\beta$ of the system before the bridge is connected, and it would be only in special instances that this $\mu\beta$ would coincide with the operating $\mu\beta$ effective while the circuit is being measured.

To illustrate further, refer to the amplifier in Fig. 5 with input and output bridges and consider the impedance the high side of the input coil faces. To get the value of $\mu\beta$ satisfying

$$Z=\frac{Z_0}{1-\mu\beta},$$

assume a small voltage across (G+K'G) and consider how this voltage will be modified in a single journey around the amplifier and feedback circuits. It will be amplified by the amplifier and attenuated by the feedback path and applied to the input bridge where, depending upon

$$\left(\frac{G_0}{G_0+G}\right)$$

a portion of it will reach the grid and, if the bridge is balanced, none of it will reach the starting point which was the points (G+K'G) to which the input coil normally connects. Thus, it is here evident that $\mu\beta=0$ with respect to

$$Z=\frac{Z_0}{1-\mu\beta}\qquad(t)$$

whereas, without the β-circuit network,

$$\mu\beta=\mu\frac{\frac{G_0}{G+G_0}}{\frac{1+K}{K}+\left(\frac{1+K'}{K'}\right)\frac{R+R_0}{G+G}}$$

from (d) with respect to amplification with feedback.

Equation (q) may be written as follows:—

$$Z=\frac{Z_0}{1-\mu\beta\left(\frac{C+Z_0}{C}\right)}=\frac{Z_0}{1-(\mu\beta)_z}\qquad(q)$$

$$Z=\frac{\frac{G_0\left[f+\frac{R_0L}{R_0+L}\right]}{G_0+f+\frac{R_0L}{R_0+L}}=Z_0}{1-\mu\left(\frac{L}{R_0+L}\right)\frac{G_0}{G_0+f+\frac{R_0L}{R_0+L}}}\qquad(q^{i})$$

$$Z=\frac{Z_0}{1-\mu\beta}\text{ for }C=\infty\qquad(q^{ii})$$

$$Z=\frac{f+R_0}{1-\mu}\text{ for }L=\infty\qquad(q^{iii})$$

$$Z=\frac{fR_0+fL+R_0L}{R_0+L-\mu L}\text{ for }G_0=\infty\qquad(q^{iv})$$

Hence, with respect to the input circuit:—

$$\text{grid voltage with feedback}=\frac{\text{grid voltage without feedback}}{1-\mu\left[\left(\frac{L}{R_0+L}\right)\frac{G_0}{G_0+f+r_0}\right]\frac{C}{Z_0+C}}$$

$$\text{impedance with feedback}=\frac{\text{impedance without feedback}}{1-\mu\left[\left(\frac{L}{R_0+L}\right)\frac{G_0}{G_0+f+r_0}\right]}$$

Showing that if $(\mu\beta)_z=1000$, for example, and $C\doteq 0$, that although feedback would divide the input impedance by practically 1000 it would not materially alter the grid voltage or amplification. On the other hand, the conjugate arrangement of Fig. 5 does change the amplification but not the impedance.

Referring to Figs. 31 and 31A,

$$Z_0=\frac{R_0(f+g')}{R_0+f+g'}\text{ output impedance without feedback}$$

$$E=V\left(\frac{f+g'}{g'}\right)=\frac{Z_0}{L+Z_0}\left(\frac{e}{1-\mu\beta}\right)$$

$$e-I_1L=E\text{ when }\frac{E}{I_1}=\frac{e}{I_1}-L=Z_0=\frac{e}{I_1}\left(\frac{Z_0}{L+Z_0}\right)\left(\frac{1}{1-\mu\beta}\right)$$

$$Z=(Z+L)\left(\frac{Z_0}{1+Z_0}\right)\frac{1}{1-\mu\beta}$$

$$Z=L\frac{1}{(1-\mu\beta)\left(\frac{L+Z_0}{Z_0}\right)-1}=\frac{1}{\frac{1-\mu\beta}{Z_0}\left(\frac{L+Z_0}{L}\right)-\frac{1}{L}}$$

$$Z=\frac{Z_0}{1-\mu\beta\frac{Z_0+L}{L}}=\frac{Z_0}{1-(\mu\beta)_z}\text{ output impedance with feedback}$$

Referring to Figs. 31 and 31A and these equations derived in connection therewith, the output impedance of the amplifier circuit of Fig. 1 is:—

$$Z_0=\frac{R_0(f+g')}{R_0+f+g'}\qquad (u)$$

$$Z=\frac{Z_0}{1-\mu\beta\frac{L+Z_0}{L}}=\frac{Z_0}{1-(\mu\beta)_z}\qquad (v)$$

where

Z_0=impedance without feedback and
Z =impedance with feedback.

Equation (v), it will be noted, is similar to (r) and evidently the statements about (r) are applicable to (v), except perhaps whether or not (v) is independent of L.

To show that (v) is independent of L, consider that

$$(\mu\beta)_z=\mu\frac{g'}{f+g+R_0}$$

whereas if β as given by (b) is multiplied by

$$\mu\left(\frac{L+Z_0}{L}\right)$$

the result is

$$\mu\frac{f'}{f+g+R_0}$$

and independent of L.

Equation (v) may be written:—

$$Z=\frac{R_0(f+g')}{R_0+f+g'(1-\mu)}\qquad (v^{i})$$

$$Z=\frac{R_0}{1-\mu}\text{ for }g'=\infty\qquad (v^{ii})$$

$$Z=R_0\text{ for }f=\infty\qquad (v^{iii})$$

$$Z=f+g'\text{ for }R_0=\infty\qquad (v^{iv})$$

With respect to the output circuit:—

$$\text{voltage with feedback}=\frac{\text{voltage without feedback}}{1-\mu\frac{g'}{g'+f+R_0}\left(\frac{L}{L+Z_0}\right)}$$

$$\text{impedance with feedback}=\frac{\text{impedance without feedback}}{1-\mu\frac{g'}{g'+f+R_0}}$$

To recapitulate with respect to configuration A, the input and output impedances with feedback are separately and respectively independent of the impedances of the input and output connecting circuits. Thus, the input impedance is independent of the input connecting circuit but does depend upon the impedance of the output connecting circuit; the output impedance is independent of the impedance of the output connecting circuit but does depend upon the impedance of the input connecting circuit. The impedance with feedback is equal to the impedance without feedback divided by $1-(\mu\beta)_z$. For the case of the input impedance, $(\mu\beta)_z$ is the value which $\mu\beta$ approaches as the impedance of the external input connecting circuit approaches infinity. Similarly, to get the output impedance with feedback, the output impedance without feedback is divided by (one minus the value which $\mu\beta$ approaches as the load impedance approaches infinity).

Referring to Figs. 32 and 32A,

$$Z_0=r_0+g'\text{ Feedback impedance without feedback}$$

Where:—

$$g'=\frac{CG_0}{C+G_0}\qquad r_0=\frac{R_0L}{R_0+L}$$

$$\beta=\frac{g'}{r_0+g'+f}$$

$$i=\frac{e}{r_0+g'+f}\left(\frac{1}{1-\mu\beta}\right)$$

$$\frac{e}{i}=f+Z=(r_0+g'+f)(1-\mu\beta)$$

$$Z=(r_0+g')(1-\mu\beta)-\mu\beta f=(r_0+g')-\mu\beta(r_0+g'+f)$$

$$Z=\left(1-\mu\beta\frac{r_0+g'+f}{r_0+g'}\right)Z_0\text{ Feedback impedance with feedback}$$

or

$$Z=(r_0+g')-\frac{\mu L}{R_0+L}g'$$

Figs. 32 and 32A and this derivation indicate a derivation of the equation for the feedback impedance of the amplifier circuit of Fig. 1, designated Z with feedback and Z_0 without feedback. This would also be a load impedance if the output were connected at f instead of L. This impedance is given by:—

$$Z=Z_0[1-(\mu\beta)_z]$$

where

$$(\mu\beta)_z=\mu\beta\left(\frac{r_0+g'+f}{r_0+g'}\right)\qquad (w)$$

This is an example of a case in which the impedance with feedback is equal to the impedance without feedback multiplied by $[1-(\mu\beta)_z]$ where $(\mu\beta)_z$ is the value with $\mu\beta$ approaches as the impedance of the connecting circuit, which in this case is f, approaches zero. As previously, the impedance with feedback is independent of f whereas the amplification with feedback depends upon f.

Considering configuration B, cut the circuit of Fig. 33 at a and b as shown and apply Thévenin's theorem. This gives Fig. 33A as the equivalent circuit for the input mesh.

Referring to Fig. 32A,

$$V=\frac{V_1}{1-\mu\beta}V_1=\left(\frac{G_0}{C+G_0+r'}\right)E$$

$$e'=\frac{C+G_0+r'}{G_0}\mu\beta V$$

$$e'=\frac{\mu\beta}{1-\mu\beta}E$$

$$r'=\frac{r(R_0+L)}{r+R_0+L}$$

whence

$$Z_0=G_0+r'$$

$$i=\frac{E}{C+G_0+r'}\left(\frac{1}{1-\mu\beta}\right)$$

$$E=i_\infty(G_0+r')-e'$$

$$E=\frac{G_0+r'}{C+G_0+r'}\left(\frac{E}{1-\mu\beta}\right)-\frac{\mu\beta}{1-\mu\beta}E$$

$$\frac{E}{i}=G_0+r'-\mu\beta(C+G_0+r')$$

$$Z=Z_0\left[1-\mu\beta\frac{Z_0+C}{Z_0}\right]=Z_0[1-(\mu\beta)_z]$$

$$Z=G_0+r'-\mu\frac{r}{r+R_0+L}G_0$$

which is independent of C.

Therefore, with simple current-current feedback, the input impedance is given by:—

$$Z=Z_0\left[1-\mu\beta\frac{Z_0+C}{Z_0}\right]=Z_0[1-(\mu\beta)_z] \qquad (x)$$

whence,

$$Z=G_0+r'-\mu\frac{r}{r+R_0+L}G_0 \qquad (x^1)$$

which is independent of C. In (x) Z_0 is given by:—

$$Z_0=G_0+r' \qquad (y)$$

It is to be observed that the impedance without feedback is multipled by $[1-(\mu\beta)_z]$ where $(\mu\beta)_z$ is the value of $\mu\beta$ for $C=0$.

If $(G_0+r')>>C$, $(\mu\beta)_z\doteq\mu\beta$ and Z approaches Z_0 multiplied by $(1-\mu\beta)$ in which case the impedance with feedback is more or less than the impedance without feedback according as the feedback is negative or positive, which will be according as $|1-\mu\beta|$ is greater or less than unity. The impedance with feedback is rotated relative to the impedance without feedback by an amount Φ_2 where

$$Z=|Z_0|\underline{|\Phi_1} \text{ and } (1-\mu\beta)=|1-\mu\beta|\underline{|\Phi_2}$$

whence

$$Z\doteq|Z_0|\;|1-\mu\beta|\;\underline{|\Phi_1+\Phi_2}$$

The only restriction on $\mu\beta$ is that it complies with Nyquist's rule for stability.

It is to be observed that if $C>>G_0$ for configuration A (in which case Z approaches

$$\left.\frac{Z_0}{1-\mu\beta}\right)$$

and $C<<G_0+r'$ for configuration B (in which case Z approaches

$$Z_0[1-\mu\beta])$$

that a modification of the input impedance with feedback as obtained with positive feedback with one configuration would in general only be obtained with negative feedback for the other configuration, the exception being in those instances where the effect of feedback is not to change the impedance or to change the angle of the impedance but not its absolute value, either of these conditions corresponding to

$$\Phi=\cos^{-1}\frac{\mu\beta}{2}$$

or boundary A in Figs. 2, 3 and 4.

Equation (x) may be written as:

$$\frac{Z}{Z_0}=1-\mu\beta\frac{C+G_0+r'}{G_0+r'}$$

It is to be noted that $(C+G_0+r')$ may be arranged to approach zero and simultaneously $\mu\beta$ may still comply with Nyquist's rule; under these circumstances the input impedance with feedback would be the same as without feedback irrespective of the values of either $\mu\beta$ or (G_0+r').

If

$$\mu\beta\frac{C+G_0+r'}{G_0+r'}=1+j_0$$

and at least $C\neq0$ or in general the conditions for instability be avoided, the impedance with feedback would always be zero and it is to be noted that this result may be obtained when $|\mu\beta|$ is large compared to unity.

If

$$0<\left|\mu\beta\frac{C+G_0+r'}{G_0+r'}\right|<2$$

the magnitude of the impedance with feedback may be more or less or the same as without feedback and in all instances the phase angle will have been altered except for the special conditions previously noted.

To recapitulate: the absolute value of the input impedance of configuration B with feedback may be made zero or it may be greater than, less than, or the same as the value without feedback; usually, but not always, its phase angle will be altered.

The output impedance of configuration B, Fig. 34, is shown as follows:

$$Z_0=R_0+\frac{r(G_0+C)}{C+G_0+r} \text{ output impedance without feedback}$$

$$V_1=-\beta E$$

$$\mu V=-\frac{\mu\beta}{1-\mu\beta}E$$

$$i=\frac{E}{L+R_0+\frac{r(G_0+C)}{r+G_0+C}}\left(\frac{1}{1-\mu\beta}\right)$$

$$Z+L=\left[L+R_0+\frac{r(G_0+C)}{r+G_0+C}\right](1-\mu\beta)$$

$$Z=R_0+\frac{r(G_0+C)}{r+G_0+C}-\mu\beta\left[L+R_0+\frac{r(G_0+C)}{r+G_0+C}\right]$$

$$\beta=\frac{\frac{r(G_0+C)}{r+G_0+C}}{L+R_0+\frac{r(G_0+C)}{r+G_0+C}}$$

$$Z=R_0+(1-\mu)\frac{r(G_0+C)}{r+G_0+C}$$

$$Z=Z_0\left[1-\mu\beta\frac{L+Z_0}{Z_0}\right]=Z_0[1-(\mu\beta)_z]$$

Thus, the output impedance of configuration B, Fig. 34, with feedback is given by:

$$Z=Z_0\left[1-\mu\beta\left(\frac{L+Z_0}{Z_0}\right)\right]=Z_0[1-(\mu\beta)_z] \qquad (z)$$

whence

$$Z = R_0 + (1-\mu)\frac{r(G_0+C)}{r+G_0+C} \qquad (z')$$

a result which is independent of L.

$$Z_0 = R_0 + \frac{r(G_0+C)}{r+G_0+C} \qquad (aa)$$

(z) is similar in form to (x) and the discussion of the properties of (x) will be pertinent to (z).

V. c. Feedback methods or configurations

Several configurations of feedback circuits have been shown in figures discussed above. Figs. 35A to 35E and Figs. 36A to 36H show a number of configurations together for comparison and contrast with each other, and indicate clearly that various configurations may be employed.

In these thirteen figures C and L are the input and load circuits, respectively, as in preceding figures; the transducer shown may be of any suitable type, as for example a vacuum tube amplifier or an electro-mechanical amplifier (i. e. a so-called mechanical amplifier); and the element Z, shown in certain of these figures, may be a two-terminal impedance and may, if desired, have a value small compared to the impedance of the input circuit C or the load circuit L with which it is associated.

Figs. 35A to 35D show voltage-voltage configurations and current-current configurations generally similar to configurations considered under the preceding heading, and Fig. 35E shows a bridge-bridge configuration such as that of Fig. 5 described above; and then various modifications or combinations of such configurations are shown in Figs. 36A to 36H.

Configuration A, (Figs. 35A and 35B), illustrates very simple feedback systems in which the wave to be fed back is obtained as a voltage.

In this illustration it can be seen that for large values of $\mu\beta$ (i. e. when $\mu\beta >> 1$) the load impedance Z_L will affect the feedback voltage in exactly the same way that it affects the voltage across the load or output. Any element affected in this way is part of the μ-circuit and hence its variations are stabilized by feedback. This means that when the voltage of the output load determines the feedback voltage in this manner the voltage across the load will always be independent of variations in the μ-circuit.

There are cases in practice where the output voltage should be held constant. In these cases the feedback signal should be obtained as a voltage across the load.

Configuration B, (Figs. 35C and 35D), illustrates very simple feedback systems in which the wave to be fed back is obtained as a current.

For large values of $\mu\beta$ (i. e. when $\mu\beta >> 1$) and when the current through the load controls the feedback signal then the current through the load will always be a constant. This method should be employed when a constant load current is desired.

Configuration C, (Fig. 35E), shows how the feedback path can be made independent of the load impedance by the use of balanced bridges. Also, when desired the bridges can be unbalanced to obtain a required dependence or to obtain a given approach toward independence.

Configurations D, E, F and G, (Figs. 36A to 36H), include several feedback systems in which a wave to be fed back is obtained as a voltage and introduced into the system as a current, or in which the wave to be fed back is obtained as a current and introduced into the system as a voltage. Various modifications of Figs. 35A to 36H can be used. By properly proportioning it is possible to obtain generalized relationship between the load voltage and load current. One expedient by which this may be accomplished is to unbalance the bridge. This generalized relationship may be made a function of frequency.

Regarding the difference between applying the fedback signal as a voltage and as a current, if configuration A is used and a fedback voltage is applied across the input, the input impedance approaches zero as $\mu\beta$ becomes larger and larger. However, if the fedback wave is introduced as a current the input impedance tends to become very high under the same conditions.

The use to which the amplifier is to be put should determine the method of obtaining the feedback signal and the method of reintroducing it into the system.

V. d. Input and output impedances of amplifiers using bridge type networks for feedback

The input impedance of amplifiers with a balanced input bridge is independent of the amount of feedback and of the impedance presented by the β-circuit-network. If the value of G_0 deviates from the value required for balancing the bridge, the input impedance follows a somewhat complicated law. Without feedback the input impedance is the same as that of the passive network. As the feedback increases to very large values the impedance approaches that of the balanced bridge and is independent of the value of the G_0 arm. For small values of ΔG_0 the derivative of Z with respect to G_0 is inversely proportional to $1-\mu\beta$. The relation is

$$\frac{dZ}{dG_0} = \frac{S}{1+S}\,\frac{(Kg+g+KS)}{(Kg+g+KS+K)(1-\mu\beta)}$$

where $G = SG_0$ and impedance looking from the bridge into the β-circuit network$=gG_0$.

Equations were also worked out for the impedance of the input when each of the other three arms of the bridge varied. All equations approach limits as the feedback becomes infinite. In each case the limit approached is the impedance which would be seen if the bridge were rebalanced by varying the G_0 arm. The derivatives expressing the path by which these limiting values are approached are not simple and are not given. The value of $\mu\beta$ used in the equations was that which existed before the arms were varied.

Similar expressions were derived for the impedance of the output bridge network. The output impedance is independent of the amount of feedback and of the impedance of the β-circuit-network in the balanced bridge case. When the R_0 arm departs from the value for a balanced bridge the output impedance approaches the value obtained with a balanced bridge if the feedback is very large. For small values of ΔR_0 and with a β-circuit-network impedance very high compared to the bridge impedance the derivative of Z with respect to R_0 is inversely proportional to $(1-\mu\beta)$. The relation is

$$\frac{dZ}{dR_0} = \frac{S^2}{(1+S)^2(1-\mu\beta)}$$

where $R_1 = SR_0$.

The derivative for the case of finite β-circuit-network impedance is more complex. It is

$$\frac{dZ}{dR_0}=\frac{S^2(K+KS+g+Kg)}{(1+S)^2(1-\mu\beta)(1+k)(K+KS+g)}$$

here again it is inversely proportional to $(1-\mu\beta)$.

The output impedance when the other three arms vary was computed assuming the β-circuit-network impedance to be very high. These equations all approach limits as the feedback becomes large. This limit is in each case the impedance which the bridge would present if it were rebalanced by varying R_0.

V. e. Theory and design of balanced bridges arranged to make the feedback path conjugate to input and output connecting circuits

Part I. Theory of balanced bridges.—With an output bridge consisting of four arms, R_0, KR_0, R and KR, and the input bridge consists of the corresponding arms G_0, $K'G_0$, G and $K'G$, the following relations hold:—

$$\frac{R_0}{KR_0}=\frac{R}{KR}$$

and

$$\frac{G_0}{K'G_0}=\frac{G}{K'G}$$

in which all symbols represent generalized impedances. It should be understood that although R_0 and R are ordinarily resistances there is no restriction on their phase angle. All of the quantities R_0, R, K, G_0, G and K' may, in general, be any realizable complex quantities and their values may change with frequency.

By the application of Kirchoff's laws, it can be shown that the voltage fed back across G_0 as a result of a voltage applied in series with R_0 is independent of the load impedance connected across the output bridge and of the terminating impedance across the input bridge so long as Equations (*q*) and (*r*) hold good. That is, so long as the bridge balance is maintained the feedback in an amplifier is independent of the impedance from which the amplifier works and of the impedance into which it works. This is the reason for using balanced bridges.

Part II. Design of resistance bridges.—Although the balanced bridges may be made up of complex impedances, for the sake of simplicity attention will be given only to the theory of the resistance bridge.

Selecting first the input bridge we find that the factors of interest are:—

(1) R_g—the impedance with which the bridge terminates the input coil.

(2) $R'\beta$—the impedance which the bridge presents to the β-circuit.

(3) I—the db transmission loss which the bridge presents to the signal voltage.

(4) β_g—the db loss which the bridge presents to the feedback voltage.

Taking these factors in turn

(1) $$R_g=\frac{(1+K')GG_0}{G+G_0}$$

(2) $$R'_\beta=\frac{K'}{1+K'}(G+G_0)$$

(3) $$I=20\log_{10}(1+K')$$

expressed as loss in db.

(4) β_g is for convenience more used in the form

$$\frac{1}{\beta_g}$$

Since the β-circuit loss is, for large values of feedback, equal to the gain of the amplifier, it is more convenient to express the quantity as a gain.

Expressed in db:—

$$\frac{1}{\beta_g}=20\log_{10}\frac{G+G_0}{G_0}$$

From the above relations the input bridge design is a straightforward calculation. Practically, the design of the bridge involves a series of compromises. Factors, such as the bulk capacitance of an input coil to ground, the input capacitance of the tube, introduce phase shift which must be kept at a minimum. If the amplifier is of high gain it is desirable to make it come as near as practicable to the theoretical minimum of noise and this sets a requirement that the input bridge loss to the signal shall be as low as is consistent with other factors. The loss of the bridge to signal can be made smaller for a given loss to the feedback by making G_0 large and K small but this soon makes G_0 too large compared to the tube reactance and so can not be carried far.

With the output bridge the controlling factors are:

(1) β_p—the db loss which the bridge presents to feedback voltage.

(2) P—the db power loss which the bridge presents to the signal energy.

Taking these factors:

(1) β_p is expressed in the form of a gain as with β_g.

Expressed in db:

$$\frac{1}{\beta_p}=20\log_{10}\left[\frac{1+K}{K}\,\frac{R_0+R}{R'_\beta}\right]$$

(2) $$P=10\log_{10}\left[\frac{W_1}{W_2}\right]$$

Where

$$\frac{W_1}{W_2}$$

is the ratio of the power expended in the load circuit before and after the insertion of the bridge. It will be assumed that if the impedance presented by the bridge is not equal to R_0 the output transformer will be made to match the bridge impedance and not the R_0 of the tube.

$$\frac{W_1}{W_2}=\left(1+\frac{R_0}{R}\right)(1+K)$$

and

$$P=10\log_{10}\left[\left(1+\frac{R_0}{R}\right)(1+K)\right]$$

Since it is of paramount importance that for a given value of β-circuit loss the output transmission loss be a minimum, the following design theory is based upon the fundamental requirement that the β-circuit loss be a minimum for a given value of P. For convenience we will introduce here two ratio terms:—

(1) $$\rho=\frac{R'_\beta}{R_0}$$

(2) $$\alpha=\frac{R}{R_0}$$

Assuming that the quantities R'_β and P are fixed we take the expression:

$$\frac{1}{\beta_p}=\frac{1+K}{K}+\frac{R_0+R}{R'_\beta}$$

and the expression for P:—

$$P=\left(1+\frac{R_0}{R}\right)(1+K)$$

and introducing the ratio terms ρ and α obtain:—

$$P=(1+K)\left(\frac{1+\alpha}{\alpha}\right)$$

$$\frac{1}{\beta_p}=\frac{1+K}{K}+\frac{1+\alpha}{\rho}$$

By substituting for K and differentiating

$$\frac{1}{\beta_p}$$

to obtain a minimum we find:—

$$K=\frac{P-1}{1+\sqrt{P/\rho}}$$

$$\alpha=\frac{\sqrt{P/\rho}+P}{\sqrt{P/\rho}(P-1)}$$

$$\frac{1}{\beta_p}=\frac{[1+\sqrt{P/\rho}]^2}{P-1}+1$$

In designing the bridges for an amplifier an equation for the gain is first set up:—

$$\text{Gain of amplifier}=20\log_{10}\frac{1}{2}\sqrt{\frac{R_g}{R_0}}-L_c+\frac{1}{\beta_g}-I+\frac{1}{\beta_p}-P$$

where:—

$20\log_{10}\frac{1}{2}\sqrt{\frac{R_g}{R_0}}$ = gain of coils = gain of amplifier with a tube having a $\mu=1$.

L_c = Loss of the coils due to departure from the ideal.

Other quantities as defined above.

The above equation assumes that there will be feedback enough so that

$$\frac{\mu}{1-\mu\beta}$$

will be practically equal to

$$\frac{1}{\beta}$$

The things we know are the R_0 of the output tube, the input and output coil constants, and the desired gain. The input bridge will be designed as already outlined. After this information is added and transposed to the left-hand side the equation becomes:—

$$\text{Gain}-20\log_{10}\frac{1}{2}\sqrt{\frac{R_g}{R_o}}+L_c-\frac{1}{\beta_g}+I=\frac{1}{\beta_p}-P$$

The most convenient way of designing the output bridge is to plot a set of curves. One sheet should be

$$\left(\frac{1}{\beta_p}-P\right)$$

plotted against ρ with P as parameter. Then there should be a sheet showing K as a function of ρ with P as parameter and a third showing α as a function of ρ with P as parameter.

Having these curves at hand and knowing the desired value of

$$\left(\frac{1}{\beta_p}-P\right)$$

and the value of ρ we can find the value of P and then from the second and third curves we can find K and α. From K and α we get the bridge constants at once.

VI. Plural Feedback

VI. a. Multiple feedback includes those cases in which a single $\mu\beta$-path includes other $\mu\beta$-paths and thus is a collection of $\mu\beta$-paths having parts in common and at the same time satisfies the condition that when the over-all system is viewed analytically in the form of an equivalent circuit it is not possible to obtain further reduction in the number of $\mu\beta$-paths surrounded by a single $\mu\beta$-path either (1) merely by combining passive impedances or (2) viewing as combined, parts of paths that are in parallel and either are composed of passive impedances or are not to be viewed as an additional $\mu\beta$-path.

VI. b. A repetition of the feed back process is considered to occur whenever a complete feedback system (single or multiple feedback) may be viewed and treated as a unit and, in addition, is used to form a new $\mu\beta$-path which path in every way is independent of the first feedback system except insofar as it is dependent upon its original over-all properties. Usually this requires conjugacies or their equivalent and the use of transformers, assuming no new unilateral devices or their equivalent to be added. Such a repetition may be repeated successively any number of times.

The limits obtainable can be extended by repetitions and multiple applications of feedback and in this respect the invention is believed to be unique.

In Fig. 1, if the feedback impedance, *f*, were composed of two impedances in parallel, the parallel combination would not be viewed herein as multiple feedback but rather as a simple feedback system involving a single β-path. Fig. 39 is another illustration of a feedback system not view as an example of multiple feedback. This Fig. 39 is described hereinafter.

Fig. 37 is viewed as an example of a vacuum tube amplifier with multiple feedback. Each of the three internal feed back loops or paths including impedance Z_1, Z_2 and Z_3 respectively, can be used to raise the gain of the stage that the path is associated with 10 db, for example, thus giving a total increase in gain of 30 db. The modulation produced by the last tube is not made more than 10 db worse, so that, with sufficient negative feedback, for the amplifier as a whole, to render the net amplifier gain unaltered, there is a net improvement of 20 db, with respect to the level of the fundamental as compared to the level of this distortion. Moreover, as the gain of each stage is raised 10 db, if desired its stability is improved instead of degraded applying the principles explained in the discussion, above, of stability of gain. Three additional paths involving Z_1', Z_2', Z_3' are used, for example, to alter the phase shift with a view to making the main combination of three stages more readily comply with Nyquist's rule for freedom from oscillation.

The amplifier of this Fig. 37 connects incoming circuit 11, terminated in reflection reducing resistance 12 and amplifier input transformer 13, with outgoing circuit 14 which includes amplifier output transformer 15. The amplifier comprises tubes 16, 17 and 18. Tubes 16 and 17 are, for example, screen grid tubes with indirectly heated cathodes, such for instance as Western Electric Company type 259-A tubes. Tube 18 is a coplanar grid tube, of the type described hereinafter in discussion of Figs. 57, 65 and 66. This tube has a control grid 19 and a space charge grid 20, and may be operated as a power tube with a high negative biasing voltage on the control grid and a high positive biasing voltage on the space charge grid, as described hereinafter for the coplanar grid tubes of the three figures just mentioned.

The output circuit of the amplifier of Fig. 37 includes a bridge circuit having as ratio arms the impedances R_0, KR_0, KR and R, and having the outgoing circuit 14 as one diagonal and including in the conjugate diagonal, or feedback diagonal, conductors 21 and 22 and feedback resistance or impedance r which is also included in the input circuit of tube 16. Between tubes 16 and 17 is an interstage coupling impedance I_1, and between tubes 17 and 18 is an interstage coupling impedance I_2. Impedances Z_1, Z_2 and Z_3 are respectively connected between the cathodes of tubes 16, 17 and 18 and the conductor 22. Impedances Z_1' and Z_2' are respectively connected between the screen grids of tubes 16 and 17 and the conductor 22; and the impedance Z_3' is connected between the space charge grid of tube 18 and the conductor 22.

There is feedback from the output circuit of the amplifier to its input circuit, through the common impedance r. If desired, this feedback can be negative feedback, with the $|\mu\beta|$ for this feedback much greater than unity and with consequent gain stabilization and distortion reduction as hereinbefore explained.

There is feedback from the plate circuit of tube 16 to its control grid circuit, through common impedance Z_1; from the plate circuit of tube 17 to its control grid circuit, through the common impedance Z_2; and from the plate circuit of tube 18 to its control grid circuit, through the common impedance Z_3. These are the three feedbacks referred to above as involving impedances Z_1, Z_2 and Z_3, respectively.

With the screen grids of tubes 16 and 17 and the space charge grid of tube 18 acting as anodes with respect to the tube cathodes, the control grids of the tubes control the currents in the circuits of these anodes, and there is (1) feedback from the screen grid circuit of tube 16 (including Z_1' to its control grid circuit, through the common impedance Z_1, (2) feedback from the screen grid circuit of tube 17 (including Z_2') to its control grid circuit, through the common impedance Z_2, and (3) feedback from the space charge grid circuit of tube 18 (including Z_3') to its control grid circuit, through the common impedance Z_3. These are the three feedbacks referred to above as involving impedances Z_1', Z_2' and Z_3', respectively. If desired, still other feedback paths in the system of Fig. 37 can be employed for operation in accordance with the principles hereinbefore explained.

Fig. 38 illustrates a repetition of the feedback process five times. In using feedback to improve modulation (i. e. to reduce modulation products relative to fundamental) the amount of improvement that a single feedback process can yield depends upon how well the specific amplifier or system employed is adapted to this purpose. For example, the amplifier corresponding to the plots of Fig. 68 gave an improvement of 60 db, yet another amplifier, with only one feedback path, corresponding to path P_1 of Fig. 38, gave 20 db improvement and then reducing the gain beyond 20 db did not result in an improvement in harmonics corresponding to each db the gain was further reduced. However, by stopping the first step of gain reduction and distortion improvement at 20 db, and employing a second feedback path P_2, as shown in Fig. 38, to secure a second step of gain reduction and distortion improvement, this second step of improvement will now be effective starting with 20 db (the end of step 1) and in this case will now carry on effectively over a range of more than 20 db additional reduction in gain. The reason for this may be seen from the following considerations. The effectiveness of the first step in improving the distortion of the circuit is limited by the distortion present in the output transformer 26. By including this source of distortion in the $\mu\beta$ path of the second feedback P_2, the second feedback process can not only continue to reduce the distortion in the tube but can also reduce that in the coil, giving a greater proportionate improvement than could result from merely increasing the feedback of P_1. This supplemental action is conditioned upon an effective conjugacy between the feedback paths P_1 and P_2. The feedback P_1 aids in enabling this conjugacy to be realized, for it gives an improved impedance on the secondary side of transformer 26, looking back toward R_0, with which to balance the bridge across which feedback P_2 is connected. This impedance is indicated in Fig. 38 as R_{02} and is, in effect, a new "R_0" for the feedback P_2 analogous to the R_0 of the tube for the feedback P_1. Thus, repetitions of the feedback process actually differ from feedback around a single path and even make it practicable to extend the advantages of feedback to a degree otherwise unattainable. In applying repetitions, it is not necessary that each step be directed toward the same objective.

In Fig. 38, the feedback process with amplifier 30 is carried out a third time, through feedback path P_3, a fourth time through feedback path P_4 and a fifth time through feedback path P_5. The amplifier 30 has bridge circuits 31 and 32 corresponding to the output and input bridges of Fig. 5. These bridges render path P_1 and transformer 26 conjugate, and render path P_1 and transformer 33 conjugate, and consequently path P_1 is conjugate to paths P_2, P_3, P_4 and P_5 (and vice versa) since they are connected to path P_1 through the transformers 26 and 33.

Similarly, bridges 34 and 35 render path P_2 conjugate to (transformers 36 and 37 and consequently to) paths P_3, P_4 and P_5 (and vice versa); bridge transformers 36 and 37 and their bridge impedances 38 and 39 render path P_3 conjugate to paths P_4 and P_5 (and vice versa); bridges 40 and 41 render path P_4 conjugate to (transformers 42 and 43 and consequently to) path P_5; and the networks 44 and 45 (which are forms of bridge circuit with their ratio arms and diagonals composed of transformer windings) render path P_5 conjugate to the incoming and outgoing circuits of the amplifier (and vice versa). The ratio arms R_0, KR_0, KR and R in bridge 31 correspond to the arms R_{02}, $K'R_{02}$, $K'R_2$ and R_2 respectively in bridge 34, and to the arms R_{03}, K_3R_{03}, K_3R_3 and R_3 respectively in bridge 40. Negative feedback amplifiers or systems which have output or input bridge circuits (or biconjugate networks) that involve bridge transformers or hybrid coils, (of which the bridge transformers 36 and 37 are examples), are claimed in my copending application Serial No. 114,390, filed December 5, 1936, entitled Wave translation systems. Fig. 14 in that application corresponds to Fig. 38 of the present application.

Repetition of the feedback process is not limited to negative feedback. For example, positive feedback for increasing gain and gain stability can be repeated to obtain greater increases than are feasible without the repetition.

Fig. 39 shows a vacuum tube amplifier circuit generally similar to that of Fig. 5, but with a β-circuit network f' in addition to the β-circuit network f which corresponds to the β-circuit network of Fig. 5. These networks are connected in a diagonal of two bridge circuits, input bridge **75** and output bridge **76**, which correspond respectively to the input and output bridges of Fig. 5; and the other diagonals of the bridges **75** and **76** are the amplifier incoming circuit **77** and the amplifier outgoing or load circuit **78** respectively. The incoming circuit includes input transformer **79** and the outgoing circuit includes output transformer **80**. The output bridge **76** has ratio arms comprising the impedances R_0, KR_0, KR and R, and if desired, this bridge may be also an attenuation equalizer, as described hereinafter in connection with Fig. 65.

Each of the networks f and f' may be, for example, a filter, an attenuation equalizer, or a phase control system the two networks having different attenuation-frequency characteristics for facilitating desired control or shaping of the amplifier gain-frequency characteristic in accordance with the principles described above; or the networks f and f' may be phase control circuits adjusted to control the phase rotation of a range of frequencies between input **77** and output **78**. The presence of f' supplies an added variable in the β-path to facilitate the design of the desired value of β and Φ.

While Fig. 39 indicates how a plurality of three terminal (or four terminal) networks can be utilized in different branches of a feedback path in a given feedback loop, on the other hand, where desired a given feedback path can be used as a common feedback path for a plurality of feedback loops, for example as a common β-circuit gain control network or attenuation equalizer for the two oppositely directed amplifiers of a 22-type repeater. Either a μ-circuit can be common to a plurality of β-circuits or β-circuit branches or a β-circuit can be common to a plurality of μ-circuits or μ-circuit branches.

Fig. 40 shows a system providing a stable loss (instead of a stable gain) for transmission from circuit **81** to circuit **82**. Resistances A and B form a T-network or artificial line whose input and output impedances are R_0. The output impedance R_0 constitutes one ratio arm of a bridge circuit whose other ratio arms are impedances KR_0, KR and R as indicated in the drawings, and the input impedance R_0 constitutes one ratio arm of a bridge circuit whose other ratio arms are impedances $K'R_0$, $K'R'$ and R'. (The input and output impedances of the artificial line need not be equal, as long as the bridges are balanced.) Thus, in Fig. 40 the artificial line has output and input bridges as has the amplifier in Fig. 5. In Fig. 40, as in Fig. 5, the outgoing line or circuit is one diagonal of the output bridge, the incoming circuit is one diagonal of the input bridge, and in the other diagonal of the two bridges is a feedback path which includes a feedback amplifier **83** shown as a vacuum tube system.

The amplifier **83** feeds back from the plate circuit of the last tube to the grid circuit of the first tube through the common impedance **85** for the purpose of stabilizing the gain of the feedback path between the input and output bridges. This feedback amplifier **83** completes the closed loop including the input and output bridges and T-network whose loss is stabilized against variations in the elements according to the principles hereinbefore explained regarding the stability of the μ-circuit. Transmission through the system of this figure is asymmetrical as regards opposite directions of propagation.

VII. Simple Resistance Circuits

In the foregoing discussion, in order to make the treatment perfectly general, complex quantities and generalized impedances have been assumed throughout. Some consideration will now be given to the simpler case in which resistances instead of generalized impedances are assumed. In this consideration, reference will be made to Figs. 41, 42, 43, 43A, 43B, 43C, 43D and 43E, which are identical in circuit configuration with certain figures of my prior application, Serial No. 298,155, filed August 8, 1928. To illustrate what this difference in treatment may involve, consider Fig. 5, the general case discussed above, and Fig. 43, where simple resistances will be assumed and which is identical with Fig. 3 of my prior application referred to. Where impedances R_0, KR_0, R, KR and R_s are generalized impedances or any two-terminal systems as in Fig. 5, the ratio of feedback voltage to plate circuit driving voltage is not necessarily constant for all frequencies, as it would be under the assumption of simple resistances made in the case of Fig. 43. In both cases, however, the feedback voltage may be made independent of the load impedance for all frequencies (Z in Fig. 43, and L in Fig. 5).

For simplicity it will be assumed that the driving voltage in the plate circuit of an amplifier is 180° out of phase with the grid voltage which produces it. For a feedback amplifier, it is sometimes desirable that there be available a voltage which is directly proportional to, and in phase with, the driving voltage in the plate circuit, and which is independent of the impedance of the work circuit. It will be seen from Fig. 41 and the derivation below, that the voltage Δe, which is the drop across resistances KR and KR_0, fulfills these three conditions.

In the figure, R_0 is a resistance (for example, the resistance of the space discharge path between the plate and the filament of a three-electrode vacuum tube); e represents, as a generator, a source of voltage e (for example, the driving voltage e produced in the discharge path by the grid voltage); and Z is an impedance (for example, the impedance of the load circuit or work circuit of the tube). Two resistances designated by their values R and KR, respectively, are connected in series across the impedance Z, K being a constant. A resistance designated by its magnitude KR_0 is connected between R_0 and the junction of KR with Z. The voltage e causes currents i_1, i_2 and i_1+i_2 to flow as indicated by the arrows.

The derivation mentioned above is as follows:

$$e=(i_1+i_2)R_0(1+K)+i_1R(1+K)$$
$$\Delta e=(i_1+i_2)R_0K+i_1RK$$
$$e=[1+K]\ [(i_1+i_2)R_0+i_1R]$$
$$\Delta e=[K]\ [(i_1+i_2)R_0+i_1R]$$
$$\frac{e}{\Delta e}=\frac{1+K}{K}$$

The circuit of Fig. 42 is like that of Fig. 41, except that a resistance R_s is bridged across KR_0 and KR. The current components flowing in the various parts of the circuit of Fig. 42 are indicated by the arrows and their accompanying letters i with the appropriate subscripts. However, in Fig. 42 the currents indicated by i_1, i_2 and i_1+i_2 and the voltage indicated by Δe are not in general of the same magnitudes as in Fig. 41. When a resistance R_s is connected across KR_0 and KR, the voltage Δe still fulfills the three conditions mentioned above. Referring to Fig. 42 this is demonstrated as follows:

$$e=(i_1+i_2+i_3)R_0+(i_1+i_2)KR_0+i_1KR+(i_1+i_3)R$$
$$e=(i_1+i_2)R_0(1+K)+i_1(1+K)R+i_3R_0+i_3R$$
$$e=[1+K]\,[(i_1+i_2)R_0+i_1R]+i_3(R_0+R)$$
$$\Delta e=[K]\,[(i_1+i_2)R_0+i_1R]=i_3R_s$$
$$\frac{e}{\Delta e}=\frac{1+K}{K}+\frac{R_0+R}{R_s}$$

Fig. 43 shows one way in which this voltage may be utilized. In this figure a three-electrode, electric space discharge amplifying device 1 has an anode-cathode space discharge path of resistance R_0, which is associated with resistances KR_0, KR, and R, and impedance Z, in the manner of resistance R in Figs. 41 and 42. The plate-filament resistance R_0 is the reciprocal of the slope of the static characteristic of plate current versus plate voltage of the discharge device at the so-called operating point, as explained in the following articles by John R. Carson in the Proceedings of the Institute of Radio Engineers: "Theory of three-element vacuum tube", vol. 7, pp. 187–200, April 1919; "The Equivalent circuit of the vacuum tube modulator", vol. 9, pp. 243–249, June 1921. (That is,

$$\frac{1}{R_0}=\frac{\partial i_p}{\partial e_p}$$

where i_p and e_p are instantaneous plate current and voltage, respectively.) In Fig. 43 the impedance Z is the primary-to-secondary impedance of output transformer 2′ which, together with circuit 3′ connected to the secondary winding of the transformer, forms the load or work circuit for the device 1. An input transformer 4′ impresses waves from circuit 5′ upon the grid of the device 1. These waves may be, for example, voice waves, or voice modulated carrier waves for transmission over carrier wave wire transmission systems or to radio transmitting antennae or waves received over such systems. The usual plate, filament, and grid batteries are shown at 6′, 7′, and 8′. A circuit corresponding to the resistance R_s of Fig. 42 and comprising the secondary winding or secondary-to-primary impedance R_{01} of transformer 4′, a resistance x and a blocking condenser 9′ in series, is connected across the resistances KR_0 and KR. The grid is connected to the junction of x and R_{01}. The condenser is a blocking condenser of large capacity, for preventing batteries 6′ and 7′ from applying steady voltage to the grid. The circuit through which battery 6′ supplies plate current for the tube comprises resistance KR_0 in series with two parallel paths, one extending through the primary winding of transformer 2′ and the other through resistances KR and R in series.

A characteristic of this circuit of Fig. 43 is that the gain is reduced by the feedback action as indicated above. To demonstrate this with specific reference to this particular circuit configuration, it is simpler to redraw the circuit to the form of Fig. 43A. From this it will be seen that R and KR, R_0 and KR_0 form the ratio arms of a balanced Wheatstone bridge, Z takes the place of the usual galvanometer and E represents a source of voltage E which is applied through a resistance of $(x+R_{01})$ that forms the input diagonal or feedback diagonal of the bridge. For the condition of balance, it is evident that there is no current in Z due to E, the driving voltage in the grid circuit.

There is some voltage V, from grid to filament. Due to V, there will be a driving voltage μV in the plate circuit, with relative polarity as shown by the plus and minus signs. The presence of this voltage is indicated in Fig. 43A by generator μV.

The effect of the two generators or voltages E and μV acting simultaneously will be considered, applying the principle of superposition in the manner indicated above.

Due to E alone, there will be a voltage from grid to filament, V_1 equal to

$$V_1=\frac{x+\frac{K}{1+K}(R_0+R)}{R_{01}+x+\frac{K}{1+K}(R_0+R)}E$$

With μ as defined in discussion of preceding figures of the drawings, due to μV acting alone, there will be a voltage from grid to filament V_2 equal to

$$V_2=\frac{\frac{R_{01}}{R_{01}+x}}{\frac{1+K}{K}+\frac{R_0+R}{x+R_{01}}}\mu V$$

or since here

$$\beta=\frac{1}{\frac{1+K}{K}+\frac{R_0+R}{x+R_{01}}}\left[\frac{R_{01}}{R_{01}+x}\right]$$

in accordance with the above definition of β,

$$V_2=\mu\beta V$$

Applying the principle of superposition,

$$V=V_1+V_2$$
$$V=V_1+\mu\beta V$$
$$V=\left(\frac{1}{1-\mu\beta}\right)V_1$$

The voltage in the grid circuit (and hence the current in the output impedance Z) is reduced from what it would be if there were no feedback, and the impedance relations were the same, and the factor by which the output current is reduced is given by

$$\frac{1}{1-\mu\beta}$$

It will now be shown that any distortion (as for example modulation product) produced in the tube is also reduced by the same factor.

In the circuit of Fig. 43B, let any primary distortion voltage produced in the tube be represented as a driving voltage D in the plate circuit. Let the voltage drop between the grid and the filament, resulting from the distortion produced in the tube be V_0. This sets up another driving voltage in the plate circuit, μV_0.

As before, with the above definitions of μ and β,

$$V_0=\beta(D+\mu V_0)$$
$$\mu V_0=\mu\beta(D+\mu V_0)$$
$$\mu V_0=\frac{\mu\beta}{1-\mu\beta}D$$
$$D+\mu V_0=\frac{D}{1-\mu\beta}$$

That is, the final, or resultant, driving distortion voltage in the plate circuit is reduced by a factor

$$\frac{1}{1-\mu\beta}$$

from what it would be without feedback.

In considering amplification, instead of adapting a μ as defined in the early part of this specification, it has been common to consider the amplification factor of a space discharge tube, as a numeric which for the sake of clarity will here be designated μ_1, but which has generally been called μ in the literature.

Then for the circuit of Fig. 43, a quantity of the form

$$\frac{1}{1-\mu\beta}$$

for illustration, would be written

$$\frac{1}{1-[-\mu_1+j0]\beta}$$

which is equal to

$$\frac{1}{1+\mu_1\beta}$$

where β, for this particular configuration is given by

$$\frac{1}{\frac{1+K}{K}+\frac{R+R_0}{x+R_{01}}}\left[\frac{R_{01}}{R_{01}+X}\right]$$

as in my prior application, Serial No. 298,155. While by the above method this result was obtained directly by substituting $-\mu_1+j0$ for μ, the same end could also have been reached, but in some instances less conveniently, by assigning polarities to the various generators of Fig. 43B and tracing through the signs of the different voltages and resulting currents as was done in my prior application Serial No. 298,155. An advantage of interpreting μ as defined above is that the question of sign or direction of flow of current depends only upon the direction or convention assumed at the outset and thereafter will be in all instances automatically taken care of by the mathematics, namely, by the complex operator μ which in turn is a complex quantity or function of complex quantities presenting itself as the complex measure of this generalized voltage-coefficient.

To illustrate this point further, if the circuit is a multi-stage amplifier with interstage coupling as in Fig. 43E, the amplification factor of the three tubes 1a, 2a, and 3a could be designated by μ_1, μ_2 and μ_3; the tube impedances by R_a, R_b and R_0; and the impedances of the interstage coupling circuits by Z_a and Z_b. Under these circumstances,

$$\mu=-\left[\mu_1\mu_2\mu_3\left(\frac{Z_a}{R_a+Z_a}\right)\frac{Z_b}{R_b+Z_b}\right]$$

and as before

$$\beta=\frac{1}{\frac{1+K}{K}+\frac{R+R_0}{X+R_{01}}}\left[\frac{R_{01}}{R_{01}+X}\right]$$

On the other hand if the penultimate stage be omitted the amplification is given by,

$$\mu=+\left[\mu_1\mu_2\left(\frac{Z_a}{R_a+Z_a}\right)\right]$$

It will be noted that due to the configuration of the elements of the circuit, if a generator is assumed in series with Z, no current will flow in the branch $x+R_{01}$. Hence, the impedance of the amplifier as seen from the load or work circuit Z is the same as it would be without feedback, and is independent of the shunt resistance $x+R_{01}$.

Now to return to a consideration of the gain of the circuit, by the feedback action the gain is stabilized with respect to variations of tubes and power. If due to any cause, the μ of the tube is reduced, which would reduce the gain, the effective voltage on the grid is increased. Similarly, variations in R_0 are stabilized. The curves of Fig. 43D, which are plotted from observed data, show that by the feedback action the load carrying capacity of the amplifier is substantially increased, the variation of gain with load is reduced, and the production of second harmonic is less by about 12 db for the particular values of μ and β employed in the test. The curves for operation without feedback are for operation with the right-hand side of condenser 9 disconnected fom the junction of R and KR and connected instead to ground through an impedance (not shown) equal to the impedance with which the feedback diagonal of the Wheatstone bridge in Fig. 43 is faced by the remainder of the bridge (i. e., equal to the combined resistance of two paths in parallel, one through KR and KR_0 in series and the other through R and R_0 in series). The resistance $(x+R_{01})$ of the feedback diagonal is so high that substituting an equal resistance across the path through KR and KR_0 would not affect the operation of the circuit. Thus, the curves for operation without feedback are for operation with the external grid-to-filament impedance and the external plate-to-filament impedance for the tube is the same as in operation with feedback.

The circuit of Fig. 43 can be modified to comprise a plurality of tubes connected in cascade, in which case μ represents the total amplification from a voltage across the grid of the first tube to a driving voltage in the plate of the last tube. For example, Fig. 43E shows one such modified circuit, with tandem connected tubes 1a, 1b and 1c replacing the tube 1 of Fig. 43. If the number of tubes be even instead of odd, and an odd number of reversals around the feedback loop is desired (as for example to reduce singing tendency), then an odd number of phase reversals in addition to those produced by the tubes themselves may be produced by the use of an interstage or other transformer with its windings poled to reverse the phase of waves passing through the transformer.

In Fig. 43A consider voltage V_3. Its value is being contributed to by both E and μV, and these voltages tend to oppose each other. It is possible, therefore, by suitable adjustment of the circuit elements, notably x, to make $V_3=0$. The value of x to accomplish this may be found as follows:

In Fig. 43C, $V=$(Voltage drop across x)$+V_3$

$\therefore V=$(Voltage drop across x) since $V_3=0$

Similarly $\mu V=$[Voltage drop across (R_0+R)]$+V_3$

$=$(Voltage drop across $R+R_0$)

The current through x must equal the current through $(R+R_0)$, for since $V_3=0$, no current flows through $(KR+KR_0)$.

$$\therefore \frac{V}{x}=\frac{\mu V}{R+R_0}$$

$$x=\frac{R+R_0}{\mu}$$

This has also been calculated according to the methods using Kirschoff's laws and the principle of superposition, with the same answer.

This means that if x is given the value

$$\frac{R+R_0}{\mu}$$

and a voltage is applied to the circuit in series with R_{01}, no voltage is produced across the feedback diagonal of the bridge. (That is, V_3 in Fig. 43A is zero.) This does not apply to distortion voltages, since these are present in the plate circuit, but not in the grid circuit; i. e., only one generator is acting. By giving x the value

$$\frac{R+R_0}{\mu}$$

we have two points (the ends of the feedback diagonal) across which the potential corresponding to the fundamental or original transmitted wave is zero, and the potential of the distortion or disturbing wave is not zero. (The required value of x can also be found or checked by trial.) Across these points we have voltages corresponding to any disturbance present in the plate circuit which is not present in the grid circuit. We can now operate on the distortion without disturbing the fundamental or original transmitted wave components.

Circuits operated in this manner form the subject of my Patent 2,003,282 granted June 4, 1935, and of my application Serial No. 411,223 filed December 3, 1929.

VIII. Wave Shaping by Feedback Path

Reference will now be made to Figs. 44 to 56, inclusive, which are identical as respects circuit configuration or physical content, to Figs. 1 to 13 of my prior application, Serial No. 439,205 above referred to.

In the amplifying system of Fig. 44, an amplifier **100** amplifies waves received over incoming line or circuit **102** and transmits the amplified waves to outgoing line or circuit **103**. The circuits **102** and **103** may be, for example, sections of a multiplex carrier telephone cable circuit, the amplifier **100** amplifying the waves of a number of carrier telephone channels simultaneously.

The circuit **102** comprises an input transformer **104** and is connected to the input side of amplifier **100** through the transformer **104** and a Wheatstone bridge **105**. The four ratio arms of the bridge comprise the four resistances **106**, **107**, **108** and **109**, respectively. The circuit **102** is connected across the arms **106** and **107** in series and forms one diagonal of the bridge. The input circuit of the amplifier is connected across the arm **109**.

The output circuit of the amplifier is connected to the outgoing or load circuit **103** through a Wheatstone bridge **110** and a blocking condenser **111**, the blocking condenser having negligibly low reactance for the waves to be amplified. An output transformer **112** is included in the output circuit. The space discharge path resistance R_0 of the last stage of the amplifier is one ratio arm of the bridge, and the circuit **103** is the output diagonal of the bridge. The four ratio arms of the bridge are designated by their resistance values R_0, KR_0, KR, and R, K being a constant.

Across the resistances KR and KR_0 in series is connected the input end of a feedback path or circuit **113** for the amplifier, the output end of this path being connected across the arms **107** and **108** of bridge **105**. Thus, the feedback path is a diagonal, (the feedback diagonal), of output bridge **110**, and is also a diagonal of the input bridge **105**. Thus, the bridge **110** connects the outgoing circuit **103** and the feedback path **113** in conjugate relation to each other, and the bridge **105** connects the incoming circuit **102** and the feedback path **113** in conjugate relation to each other. Consequently, the feedback action and the operation of the amplifying system are independent of the impedance of the incoming circuit and the impedance of the load circuit, and moreover cannot affect the impedances which face the incoming circuit and the load circuit at the amplifying system.

This amplifier system is of the general type disclosed in my copending application, Serial No. 298,155, referred to above. Fig. 3 of that application shows a single stage amplifier of this general type, and Fig. 3E of that application shows a three-stage amplifier of this general type. As explained in that application, by making the phase of the feedback such that the distortion waves which appear in the driving voltage generated in the space discharge path of the last stage of the amplifier and are transmitted through the feedback path and the amplifier, return in such phase as to reduce their amplitude the transmission distortion in the amplifier is reduced, the load capacity of the amplifier is increased, and the operation of the amplifier is stabilized, as for example, against gain changes which tend to result from variations of tubes or tube energizing power that occur in the system. This system increases the load carrying capacity of the space discharge tubes (1) not only by obtaining an increase in load capacity of very great importance by suppression of distortion components of frequencies other than the fundamental frequencies and thereby permitting the tubes to operate over a larger range of their grid-voltage plate-current characteristics but also (2) by obtaining a second increase of very great importance in the load capacity by feedback of fundamental waves in such a way as to control gain in a desired manner, as for example, to prevent undesired lowering of gain, for the fundamental waves.

In connection with this latter increase, it should be noted that the amplifying system of the invention comprises means for correcting for distortion caused by improper degree of amplification of fundamental waves, as for example, caused by amplification of a fundamental wave of a given frequency different amounts for different input amplitudes, or as for example, caused by amplification of two waves of respectively different fundamental frequencies, different amounts respectively. For example, if a wave of a given fundamental frequency is amplified in amplifier **100** to a degree less than, say, the normal amplification for the amplifier **100**, then for that frequency the feedback voltage across the feedback diagonal of the input bridge **105** tends to be lower than normal. As a result the tendency toward lower than normal gain of the system for the fundamental wave of the given frequency is checked. Similarly, if the given frequency is amplified to a degree greater, instead of less, than normal in the amplifier **100**, for that frequency the voltage fed back to the input bridge **105** tends to be higher than normal; and as a result the tendency toward higher than normal gain of the system for the fundamental wave of the given frequency is checked. The system compensates for too low or too high gain for fundamental waves, at the same time that it suppresses components of frequencies other than fundamental frequencies.

Notwithstanding the fact that impedances R, R_0, KR_0 and KR have in certain cases been referred to above as resistances, when desired they can be generalized impedances of any other suitable character (proper provision being made, of course, for the necessary supply of steady potential to the plates and grids of the tubes, for instance as indicated in the drawings). For example, while they can be resistances and K can be a numeric, it is not necessary that this always be the case and (as indicated above) these five quantities can, when desired, be complex quantities or have any suitable values, as long as

$$\frac{R}{KR}=\frac{R_0}{KR_0}$$

when the bridge of which they form the ratio arms is to be balanced; and similarly, notwithstanding references above to impedances x, R_{01}, R_A and R_T as resistances, they can, when desired have their impedance values complex quantities or any suitable values. The function of K in addition to being a complex measure may have different values for different frequencies. Also where reference has been made to Wheatstone bridges shown as comprised of resistance arms, the bridge in each instance need not be a resistance bridge but generalized impedances may be assumed in place of the resistance arms.

The amplifier **100** is shown as a three-stage amplifier in Fig. 56 described hereinafter. (As noted above, Fig. 56 is a circuit diagram of the system shown schematically in Fig. 44). The amplifier may have any number of stages, but the circuit should be proportioned such that the phase shift of the circuit from the space path of the last stage of amplifier **100** through the feedback path and the amplifier **100** back to the space path of the last stage, (including any phase shift in the feedback path and any phase shift in either the tubes or other portions of the amplifier), is not such as to cause the amplifier to sing. One of the factors which affects and controls the phase shift around the amplifier and feedback circuit is the number of stages. If the number of vacuum tube stages in that circuit be an odd number and the phase shift in the tubes be the only phase shift produced in that circuit, then the phase shift of that circuit will be 180°, or the distortion waves generated in the space path of each tube will return to that point in phase exactly opposite to their original phase.

As indicated above, by making $|\mu\beta|>>1$ the gain is set by the feedback circuit and in an opposite sense (i. e., the β-circuit working backwardly or inversely so that increasing the loss in it actually increases the amplifier gain).

As was pointed out above under section II. *a.* "Stability of amplification with respect to variations in μ and β", at the end of that discussion, the amplification ratio with feedback ($|A_F|$) approaches

$$\left(\frac{1}{-\beta}\right)$$

if $\mu\beta>>1$, and if the argument of $\mu\beta$ is $\pm 90°$ the approach to

$$\left(\frac{1}{-\beta}\right)$$

is improved by an amount corresponding to twice the db reduction in gain due to feedback. Assuming

$$\mu\beta=10\underline{|\pm 90^\circ}$$

this approach will be 10 times as precise as if

$$\mu\beta=10\underline{|0^\circ \text{ or } \pm 180^\circ}$$

It will be shown that if the absolute value of $\mu\beta$, (that is $|\mu\beta|$) is

$$\frac{1}{2\cos\Phi}$$

the amplification ratio $|A_F|$ is exactly

$$\left|\frac{-1}{\beta}\right|$$

Thus let

$$|A_F|=\frac{|\mu|}{\sqrt{1-2|\mu\beta|\cos\Phi+|\overline{\mu\beta}|^2}}=\frac{-1}{|\beta|}$$

then

$$\frac{|\overline{\mu\beta}|^2}{1-2|\mu\beta|\cos\Phi+|\overline{\mu\beta}|^2}=1$$

$$1=2|\mu\beta|\cos\Phi$$

$$|\mu\beta|=\frac{1}{2\cdot\cos\Phi}$$

which is the condition necessary that the amplification ratio, $|A_F|$, equal

$$-\frac{|1|}{|\beta|}$$

It is significant to note that in the case of β-circuit equalization if $|\mu\beta|$ follows the above law the amplification ratio of the system will be an exact inverse reproduction of the characteristic of the β-circuit network. Variations in μ may arise from change of gain with tubes, voltages E_B, E_S, E_C, and E_A (on the plates, shield grids, control grids and filaments, respectively), output power, frequency, phase shift, harmonic production, etc. However, when tubes, not shown, are inserted in the β-circuit, stabilization as regards harmonic production in such tubes would not be produced. Experimental measurements have shown stabilization even with

$$\mu\beta=10{,}000\underline{|310^\circ}, \text{ and even with } \mu\beta=10\underline{|340^\circ}$$

These angles include the phase shift of the tubes as well as the remainder of the circuit.

In the feedback path **113** in Fig. 56 are shown attenuation equalizing networks **121** and **122**, a gain control pad **123** and the gain regulating attenuation equalizer **124**. Equalizer **121** is a basic equalizer, for example, for equalizing the attenuation of 22 miles of cable at a temperature of 2.5° C. where the amplifier system is to provide the required amplification for approximately 25 miles of the multiplex carrier cable in which the amplifier system is connected and where that temperature is to be considered the lowest average temperature which that section of the cable assumes at any given time. Equalizer **122** is a building out equalizer to be used where required to take care of variations in cables and spans. The equalizer **122** is adaptable for equalizing attenuation of 2, 4 or 6 miles of the cable at the temperature just mentioned. The gain control pad **123** is a pure resistance network for introducing attenuation in the feedback circuit, to control the gain of the amplifier system as described hereinafter. The gain regulating transmission equalizer **124** is adjustable to compensate, as described hereinafter, for attenuation changes which vary with frequency, in an assigned length of the cable in which the amplifier system is connected. These changes may be, for example, attenuation changes caused primarily by temperature changes in the length of cable to which this equalizer is assigned. This length may be, for example, approximately 50 miles, so that such gain regulating attenuation equalizers are required at only alternate repeaters in the cable.

With proper adjustment of the complete circuit to avoid singing and with values of $\mu\beta$ sufficiently great the distortion suppression sought is obtained and a network located in the feedback circuit contributes to the transmission characteristics of the amplifying system a transmission characteristic which is the inverse of the transmission characteristic of the network, or in other words produces in the output circuit an effect upon the transmission which is the inverse of the effect that the network itself produces on transmission passing through the network. For example, a network such as the gain control pad 123, which introduces attenuation in the feedback path, increases the gain of the amplifier system; a low pass filter (not shown) in the feedback path acts as a high pass filter in the amplifier 100 or in the incoming circuit 102 or in the outgoing circuit 103; a high pass filter (not shown) in the feedback circuit acts as a low pass filter in the main transmission circuit; a band pass filter (not shown) in the feedback circuit acts as a band suppression filter in the main transmission circuit; etc.

The frequency range of the waves to be amplified by the amplifier may be, for example, from 4 kilocycles to 40 kilocycles.

For this frequency range, Figs. 45 to 55 show transmission characteristics of (1) the section of cable circuit assigned to the amplifier, (2) the amplifier without the networks 121 to 124, and (3) the networks 121 to 124, respectively, and show the effects of the networks on the transmission characteristics of the amplifier. Figs. 47, 48 and 50 show the attenuation characteristics 131, 132 and 133 of networks 121, 122 and 123, respectively; Fig. 52 shows the attenuation characteristic 134 of the network 124 when adjusted for the lowest cable temperature for which it is designed; and Fig. 54 shows by curve 134′ the change in the attenuation characteristic of the network 124 from its characteristic as shown in Fig. 52 when the network is operated from the setting for the lowest cable temperature to the setting for the highest cable temperature for which it is designed. This attenuation change in the network 124 is equal to the attenuation change which the temperature change produces in the section of cable for which the repeater is designed to give compensation for attenuation changes caused by cable temperature changes.

In Figs. 45, 46, 49, 51, 53, and 55, the zero or reference transmission level is taken as the transmission level at the output of the repeater preceding the amplifier shown in the drawings and the axis of coordinates intersect at the point marked zero on the scale of levels. Thus, in Fig. 45 the line 140, represents the transmission level at the output of the preceding repeater in the cable. In traversing the circuit from the receiving repeater to the amplifier shown in the drawings the transmission lowers its level from the level indicated by line 140 to the level indicated by curve 141 of Fig. 45. Curve 141 represents the transmission level (in db below zero level) at the input of the amplifier shown in the drawings. This curve shows that the cable attenuation increases with frequency.

Fig. 46 shows that if the networks 121 to 124 were absent from the feedback circuit (i. e., if the output bridge were connected to the input bridge through the feedback path without the networks 121 to 124), in passing through the amplifier the transmission would rise from the level indicated by curve 141 to the level indicated by curve 142, which represents the transmission level that would then obtain at the output of the amplifier. The difference between the transmission level indicated by curve 141 and that indicated by curve 142 represents the transmission gain that the amplifier would produce without the networks 121 to 124.

Fig. 49 shows that by the introduction of network 121 the transmission level at the output of the amplifier is raised from the level given by curve 142 to the level given by curve 143. The difference between the level in the curve given by curve 142 and that indicated by curve 143 represents the gain increase obtained for the amplifier by the introduction of network 121. Over the frequency range from 4 to 40 kilocycles this gain increase is proportional to the attenuation indicated by curve 131 of Fig. 47. Thus, the transmission loss of network 121 has produced an inverse effect upon transmission through the amplifier system. In fact, at each frequency the gain increase is equal to the loss indicated for that frequency by curve 131.

Fig. 49 shows that by the introduction of network 122 the transmission level at the output of the amplifier is raised from the level given by curve 143 to the level given by curve 144. The difference between these levels represents the amplifier gain produced by the equal transmission loss introduced in the feedback path by network 122. The curve 144 shows that the networks 121 and 122 have so raised the amplifier gain over the frequency range to be transmitted as to equalize the transmission or give uniform transmission over the frequency range, for the cable temperature of 2.5° C. (The curves of Figs. 48 to 55 are for the condition in which the network 122 is adjusted to equalize for four miles of cable at 2.5° C. As described hereinafter in connection with Fig. 56, this network is adjustable to adapt it to equalize for different lengths of cable).

Fig. 51 shows that by introduction of gain control pad 123 the transmission level at the output of the amplifier was raised from the level indicated by curve 144 to the level indicated by curve 145, or in other words, that by introducing in the feedback path a transmission loss uniform over a given frequency range, the amplifier gain at each frequency range is increased by an amount equal to the loss introduced in the feedback path at each frequency.

Fig. 53 shows that when the gain control equalizing network 124 is introduced in the feedback circuit, with the network in its setting for the lowest cable temperature, the transmission level at the output of the amplifier is raised from the level indicated by curve 146 to the level indicated by curve 147, or in other words, that the gain of the amplifier is increased at each frequency by an amount equal to the attenuation of the network at that frequency. This gain increase is uniform over the frequency range to be transmitted, since the attenuation of the network, in its setting for the lowest cable temperature, is uniform over the frequency range. The level shown by curve 147 is the same as the level shown by curve 140, or in other words, with networks 121, 122, 123 and 124 introduced, and with network 124 in its setting for the lowest cable temperature, the amplifier gain is just sufficient to overcome the attenuation of the section of cable assigned to the amplifier.

Fig. 55 shows that by changing the setting of the network 124 from its setting for the lowest cable temperature to a setting for the highest cable temperature for which it is designed, the amplifier gain at each frequency is increased by an amount equal to the additional loss (as given by curve 134' of Fig. 54) which the change of setting introduces in the feed back path at that frequency. If the network remained in its setting for the lowest cable temperature under the condition of the highest cable temperature, the output of the amplifier would decrease from the level indicated by curve 147 to the level indicated by curve 148. The change in the network setting prevents this decrease, and maintains the amplifier output level indicated by curve 147. The difference between the level given by curve 147 and the level given by curve 148 indicates the attenuation change that the temperature variations produce in the section of the cable for which the amplifier is designed to give compensation for attenuation changes. The setting of the network 124 can be changed to make it compensate for an attenuation change in the cable resulting from cable temperature changes less than the maximum, in a manner similar to that in which it compensates for the attenuation change caused by the maximum temperature change.

IX. Illustrative Examples of Circuits in Detail

Several examples will now be given of amplifiers, some of which have been used under conditions simulating those encountered in commercial multiplex carrier service, as illustrative of the detail design of circuits with which to practice the present invention.

The circuit diagram of the amplifier system shown schematically in Fig. 44 may be, for example, as shown in Fig. 56. Amplifier 100 is shown as a three-stage amplifier, the first two stages comprise, for example, two shield grid or screen grid thermionic vacuum tubes 151 and 152 of the equipotential cathode or heater type. The third stage comprises, for example, a three-electrode tube 153 with filamentary cathode. The tubes 151 and 152 may be Western Electric Company type 245-A vacuum tubes, and the tube 153 may be a Western Electric Company type 205-E tube.

These three tubes have a common plate battery 155 and a filament heating battery 156 sending heating current through the filaments of the three tubes in series.

Plate current for tube 153 passes from battery 155 through choke coils 157 and 158 and the primary winding of output transformer 112 to the plate of the tube. Condensers 159 and 111 cooperate with the coils 157 and 158 to prevent voltage fluctuations in the battery circuit from reaching the plate and to prevent the A. C. plate voltage from causing feed back in the amplifier through the common battery circuit. The blocking condenser 111 prevents voltage from battery 155 from reaching resistances KR_0 and KR and is a by-pass condenser for waves of the frequency to be amplified.

Plate current for tube 151 passes from battery 155 to the plate through a resistance 160 and an interstage coupling impedance comprising a choke coil 161, and a resistance 162. The resistances 160 and 162 may have values of, for example, 20,000 ohms, and 10,000 ohms, respectively. The resistance 160 and a condenser 163 form a frequency selective circuit for preventing voltage fluctuations in the battery circuit from reaching the plate and for preventing the waves in the A. C. output circuit of the tube from passing to the portions of the plate battery circuit common to a plurality of tubes of the amplifier. The condenser 163 is a by-pass condenser for waves of the frequency to be amplified. The resistance 162 tends to reduce phase shift in the amplifier (especially at frequencies below 4 kilocycles) which tends to result from the shunting of the transmission path by portions of the space current supply circuit. The inductance 161 resonates with its self-capacity 161' (i. e., forms an antiresonant circuit in series with 162) at approximately 4 kilocycles, so that it acts as a very small capacity substantially throughout the transmission range of 4 to 40 kilocycles. Thus the coupling impedance 161, 162 is high and fairly uniform throughout this frequency range.

Plate current for tube 152 passes from battery 155 through a resistance 170, a choke coil 171 and a resistance 172 to the plate of the tube. Each of the resistances 170 and 172 may have a value of, for example, 12,000 ohms. Elements 70, 170, 171, 172 to 173 function in connection with tube 152 in the manner in which elements 160 to 163 function in connection with tube 151.

Battery 155 supplies steady biasing potentials for the shield grids of tubes 151 and 152 through the frequency selective networks 175 and 185, respectively. The network 175 is shown as consisting of two series arms 176 and 177 of resistance and two shunt arms 178 and 179 of capacity. The network 185 is shown as consisting of two series arms 186 and 187 of resistance and two shunt arms 188 and 189 of capacity. These networks prevent voltage fluctuations from the plate battery circuit and voltage waves from the A. C. output circuits of tubes 151 and 152 from reaching the shield grids. The resistances 176 and 177 adjust the voltage applied from battery 155 to the shield grid of tube 151 to the proper operating value; and the resistances 186 and 187 adjust the voltage applied from battery 155 to the shield grid of tube 152 to the proper operating value.

Grid biasing batteries 210, 220 and 230 are provided for the tubes 151, 152 and 153, respectively. Battery 210 supplies steady biasing potential to the control grid of tube 151 through a resistance 211 and a grid leak resistance 212. Resistances 211 and 212 cooperate with a condenser 213 to form a frequency selective network for preventing voltage fluctuations from the battery 210 from reaching the control grid of tube 151 and to cause the signal waves in the input circuit of the tube 151 to be by-passed through condenser 213 around a grid bias voltage supply circuit. The condenser 213 is a by-pass condenser for waves of the frequencies to be amplified. A blocking condenser 214 prevents flow of current from battery 210 through resistance 106 or the secondary winding of transformer 104.

Elements 220 to 224 function in connection with tube 152, and elements 230 to 234 in connection with tube 153, in the manner in which elements 210 to 214 function in connection with tube 151. Moreover condensers 224 and 234 prevent battery 155 from applying potential to the grids of tubes 152 and 153.

A blocking condenser 235 prevents passage of current from battery 155 through resistance R. The condensers 214, 224, 234 and 235 are of negligibly low reactance for the frequencies of the waves to be amplified.

The following considerations indicate how, as stated above, the equalizer 121 in the β-circuit causes the amplifier to discriminate in favor of the lower frequency channels of a large number of channels (transmitted through the cable or other circuit in which the amplifier is connected) as regards the lowering of the modulation level produced in the amplifier. The modulation level without an equalizer in the β-circuit may be taken as independent of frequency, and as, for example, 75 db below the signal output. Then with an equalizer in the β-circuit it is 75 db at 40 kilocycles and less at other frequencies (frequency referring to the frequency of the modulation product itself and not to the frequencies of the signals producing the product, it being independent of the signal frequencies). Assuming the attenuation characteristic of the equalizer and also that of the cable is linear with frequency and slopes from zero to 21 db over the frequency band, then with the equalizer in the β-circuit, the harmonic or modulation level will be 75 db (below the signal output) at 40 kilocycles and 96 db (below the signal output) at 4 kilocycles, and the level will be linear with respect to frequency. As indicated hereinabove, this is highly advantageous for a large number of channels because in this case the lower channels are subject to considerably more interference than the upper channels.

Moreover, when the noise level spectrum of the cable is flat or independent of frequency, or is made so (as for example, by shielding the cable so well that the only source of noise is thermal agitation), and all the lower channels are sent out toward the amplifier at a lower level than the upper channels so that the signal or side band levels of the channels are all equal at the input to each repeater rather than at the output of the repeater or amplifier, there is a double advantage in using an equalizer in the β-circuit and, for the example given, modulation in the amplifier is about 21 db less troublesome than would be the case if all channels were at the same level at the output and the equalizer were in front of the amplifier and were the complement of the line. This represents a power economy in output capacity of the last tube of over 100:1 if the capacity is set by modulation considerations as is usually the case in current carrier engineering practice.

Where desired, for example, for a case in which the number of channels is such as to require a transmission frequency range of only the order of magnitude mentioned above (4 to 40 kilocycles) and the noise level spectrum is not substantially independent of frequency, the lower frequency channels may be sent out toward the amplifier at a lower level than the higher frequency channels, but so that the side band levels at the input to each repeater are somewhat lower for the high frequency channels than for the low frequency channels, and at the output of the repeater are somewhat higher for the high frequency channels than for the low frequency channels.

Fig. 57 (referred to above) shows the circuit of a negative feedback amplifier 240 comprising vacuum tubes 241, 242 and 243 in cascade connection, for amplifying waves of a frequency range extending from 4 kilocycles to 40 kilocycles received over incoming line or circuit 238 and transmitting the amplified waves to outgoing line or circuit 239. The circuits 238 and 239 may be, for example, sections of a non-loaded multiplex carrier telephone cable circuit, the amplifier amplifying simultaneously a number of carrier telephone and/or carrier telegraph messages of channels extending over the 4 kilocycles to 40 kilocycles range.

The incoming circuit comprises an input transformer 244 and is connected to the input side of the amplifier through a bridge circuit 245. The four ratio arms of the bridge comprise the four resistances 246, 247, 248 and 249 respectively, and in the arm containing resistance 246 is also an adjustable phase correcting condenser 246′ which balances the effective input capacity of tube 241 and reduces the phase shifts around the feedback loop to values favorable for avoiding danger of the amplifier tending to be self-oscillatory at high frequencies. The circuit 238 is connected across two of the bridge arms in series and forms one diagonal of the bridge. The input circuit of the amplifier is connected across the arm 249.

The output circuit of the amplifier is connected to the outgoing or load circuit 239 through a bridge circuit 250 and a stopping condenser 251, the stopping condenser having negligibly low reactance for the waves to be amplified. An output transformer 252 is included in the outgoing circuit. The ratio arms of the bridge are resistances R_0, KR_0, KR and R, K being a constant and R_0 being the space path resistance of tube 243. A stopping condenser 259 is included in the arm that contains resistance R. The circuit 239 is the output diagonal of the bridge.

Across the resistances KR and KR_0 in series is connected the input end of a feedback path for the amplifier, comprising conductor or feedback lead 253 and ground, the output end of this path being connected across the arms 247 and 248 of bridge 245. Thus, the feedback path is a diagonal (the feedback diagonal) of output bridge 250, and is also a diagonal of the input bridge 245, as in the case of Fig. 5, for example, described above.

Tubes 241 and 242 are heater type screen grid tubes of high amplification factor (Western Electric Company type 245-A tubes). Tube 243 is a coplanar grid tube of the general type disclosed by H. A. Pidgeon and J. O. McNally in their copending application Serial No. 368,647, filed June 5, 1929, or in their copending application Serial No. 542,252, filed June 5, 1931, (which have now patented as No. 1,923,686, August 22, 1933, and No. 1,920,274, August 1, 1933, respectively), or in their paper published in the Proceedings of the Institute of Radio Engineers, vol. 18, pages 226 to 293, February, 1930. Such a tube has two grids, each active elementary area on either grid being close to a corresponding active area on the other grid and being at substantially the same location as that corresponding area with respect to the cathode and the anode or plate. For example, each grid may have its active area in the same surface, for instance the same plane or cylindrical surface, as the active surface of the other grid. As brought out in the above mentioned disclosures of Pidgeon and McNally, a vacuum tube having a space charge grid in coplanar relation with a control grid is especially adapted for operation as a power tube by employing a high value of control grid negative biasing potential and a control grid input voltage wave of large amplitude (i. e., a large grid swing) and a high positive biasing voltage on the space charge grid. Tube 243 is a Western Electric Company type 281-A tube operated in that way, with grid 254 serving as a control grid receiving the signal voltage transmitted from tube 242, and with grid 255 serving as a space charge grid. Grid 254 is maintained at negative potential by a negative biasing voltage applied from a 70-volt battery or source **256** through resistance **257**. Grid **255** is maintained positive by a positive biasing voltage applied from a 70-volt battery or source **258** across which is shunted a by-pass condenser **258'**.

Tubes **241**, **242** and **243** have a common 130-volt plate battery or source **260** and a 24-volt filament heating battery or source **261** sending heating currents through the filaments of the three tubes in series, the positive pole of the filament heating battery being shown connected to the negative pole of the plate battery.

Plate current for tube **243** passes from battery **260** through choke coil **262** and the primary winding of output transformer **252** to the plate of the tube. This direct current is prevented from reaching resistance R by the stopping condenser **259** and is prevented from reaching resistance KR_0 by condenser **251** which is a by-pass condenser for waves of the frequency to be amplified and which cooperates with the choke coil **262** and a condenser **270** to prevent voltage fluctuations in the battery circuit from reaching the plate and to prevent the alternating current plate voltage from causing feedback in the amplifier through the common battery circuit.

Plate current for tube **241** passes from battery **260** to the plate through a resistance **263** and an interstage coupling impedance or choke coil **264**. The resistance **263** and a condenser **265**, in conjunction with the condenser **270**, form a frequency selective circuit for preventing voltage fluctuations in the battery circuit from reaching the plate and for preventing the waves in the alternating current output circuit of the tube from passing to portions of the plate battery circuit common to a plurality of tubes of the amplifier. The condenser **265** is a by-pass condenser for waves of the frequency to be amplified.

Plate current for tube **242** passes from battery **260** through a resistance **266** and a choke coil **267** to the plate of the tube. Elements **266** to **268** function in connection with tube **242** in the manner in which elements **263** to **265** function in connection with tube **241**.

Battery **260** supplies steady positive biasing potential for the screen grid of tube **241** through a frequency selective network comprising a series resistance arm **271** and shunt capacity arms **272** and **270**, and supplies steady positive biasing potential for the screen grid of tube **242** through a frequency selective network comprising a series resistance arm **273** and shunt capacity arms **274** and **270**. These networks prevent voltage variations in the plate battery circuit from reaching the screen grids, and prevent waves in the screen grid circuits from passing to portions of the plate battery circuit common to a plurality of tubes of the amplifier. The resistances **271** and **273** adjust the voltages applied from battery **260** to the screen grids of tubes **241** and **242**, respectively, to the proper operating values, and assist in maintaining the amplification at proper values at frequencies in the lower portion of the utilized frequency range.

Negative biasing potentials for the control grids of tubes **241** and **242** are obtained from the voltages across resistors **275** and **276**, respectively, that result from flow of the space currents of the respective tubes through those resistors. The voltage across resistor **275** reaches the control grid of tube **241** through the bridge **245** and conductor **227** and the secondary winding of transformer **244**. The voltage across resistor **276** reaches the control grid of tube **242** through grid leak resistor **278**. Condensers **279** and **280** in the interstage coupling circuits are stopping condensers, of negligibly low reactance for the frequencies of the waves to be transmitted.

Resistances **275** and **276** are not by-passed for alternating currents and hence form a common impedance between the plate circuit and grid circuit of their respective tubes and produce negative feedback stabilizing the gain introduced by these two tubes **241** and **242**.

From Fig. 58, which, as noted above, is a polar plot of $\mu\beta$ for the amplifier of Fig. 57, the values of $|\mu\beta|$ for various frequencies can be seen. For example, for frequencies in the neighborhood of 10 kilocycles $|\mu\beta|$ is approximately 100.

Fig. 59 shows the gain-frequency characteristic of this amplifier of Fig. 57 without feedback, and with two different adjustments of the circuit as regards amounts of negative feedback, the curve labeled Feedback 1 corresponding to one of these two adjustments and the curve labeled Feedback 2 corresponding to the other of these two adjustments. The three curves show how the gain varies with frequency as the amount of feedback is changed, and make it readily apparent that the gain variation is far less in the feedback condition. The gain includes that due to the input and output transformers. The line labeled Operating range in Fig. 59 extends from 4 kilocycles to 40 kilocycles to indicate that the amplifier is designed for operation over that frequency range.

Fig. 60 shows how negative feedback affects variation of the gain of this amplifier of Fig. 57 caused by variation of plate battery voltage. The curves of this figure were taken at 10 kilocycles, the upper curve with no feedback and the lower one with negative feedback. They show that the gain variations are reduced by the negative feedback—by an amount corresponding to the gain reduction at the frequency of measurement. This is 48 db at 10 kilocycles as shown in the curve labeled Feedback 1 in Fig. 59.

Fig. 61 gives curves obtained with this amplifier of Fig. 57, showing the effect of negative feedback upon second harmonic and third harmonic, for the same outputs of fundamental (in milliamperes or watts) with feedback as without feedback. The curves taken without feedback are labeled No feedback. The curves labeled Feedback 1 are taken with the same adjustment of the circuit as was used for the curve labeled Feedback 1 in Fig. 59. The frequency of the fundamental wave used in taking the curves of the second harmonics was 7.5 kilocycles; and the fundamental frequency used in taking the third harmonic curves was 5 kilocycles. It can be seen that up to about 35 milliamperes output of fundamental, the negative feedback reduces the ratio of harmonics to fundamental by approximately the amount of the gain reduction. (The gain reductions at 7.5 kilocycles and at 5 kilocycles can be read from the curves labeled No feedback and Feedback 1 in Fig. 59.)

Fig. 62 gives the gain-load characteristics of the amplifier of Fig. 57 with and without feedback. The curve labeled Feedback 1 was taken at 10 kilocycles and with the same circuit adjustment as the curves labeled Feedback 1 in Fig. 61. The curves of Fig. 62 show that the negative feedback greatly reduces the change of gain with level. The output current is given in milliamperes into a fixed resistance as usual in such curves.

Fig. 63 shows curves representative of the gain stability of a single amplifier such as that of Fig. 57 as determined by an average of the variations in 69 such amplifiers connected in tandem (in a non-loaded carrier telephone cable circuit approximately 1700 miles long.) In comparison of this figure with Fig. 58, which gives a plot of $\mu\beta$ for a large range of frequencies (for the amplifier of Fig. 57 which is representative of each of the 69 amplifiers in tandem) and the stability boundaries in Figs. 2, 3 and 4, it will be noted that there are four frequencies for which $|\mu\beta|$ and Φ satisfy boundary "C" and result in perfect stability. Frequencies between approximately 200 cycles and 1300 cycles should all decrease in gain as the μ of the amplifier is increased; while frequencies between approximately 1300 cycles and 30,000 cycles should increase in gain with an increase in μ, and likewise frequencies between approximately 30,000 cycles and 200,000 cycles should decrease with any increase in μ. Fig. 63 demonstrates that the experimental evidence supports the theoretical deductions.

To illustrate how the amplifier of Fig. 57 operates on each of the other boundaries, D, E, G and I, these boundary lines have been drawn in or indicated on the polar plot of Fig. 58. From reference to that figure it is seen that the amplifier operates on boundary D at a frequency of approximately 200 cycles; on boundary E at two frequencies, respectively, just below 2 kilocycles and just above 30 kilocycles; on boundary G at approximately 12 kilocycles; and on boundary I at four frequencies, respectively, about 280 cycles, approximately 1900 cycles, approximately 31 kilocycles and approximately 180 kilocycles. Boundaries B and H are also indicated on this same figure but the measurements were not carried far enough to determine the relationship of the operating characteristic to either of these boundaries. It will be apparent from the curve however that with the proper circuit constants the amplifier could be made to operate at some one or more frequencies desired on either of these latter boundaries.

Fig. 64 shows curves of stability of gain with respect to varying plate voltage plotted as abscissae. Within the limits indicated on this figure, it is found that the decibel change in gain without feedback is substantially proportional to change in anode voltage, so that the curves on this plot should have substantially the same shape as those in Fig. 63. It will be seen that this is the case, considering that a more open scale is used for ordinates in Fig. 64.

The plots in Fig. 64 are also from data measured on 69 amplifiers in tandem, as in the case of Fig. 63, and show how much the gain varies with feedback for a given change in plate voltage which, as stated, is approximately proportional to the change in gain without feedback. A comparison of this Fig. 64 with the polar plot of Fig. 58 and the conditions for boundary "C" in Figs. 2, 3 and 4, shows the variations in stability with various values of $\mu\beta$. From Fig. 58 it can be seen that there are four frequencies for which the stability is perfect. Fig. 63 shows two of the cross-over points approximately 1300 cycles and 25,000 cycles.

The system of Fig. 57 is claimed in my application Serial No. 173,749, filed November 10, 1937, entitled Wave amplification, which is a division of the present application.

Fig. 65 shows a circuit employing negative feedback, in which an amplifier comprising vacuum tubes **301**, **302** and **303** in cascade connection, amplifies waves received over incoming line or circuit **304** and transmits the amplified waves to outgoing line or circuit **305**. The circuits **304** and **305** may be, for example, sections of a one-way path of a two-way, four-wire carrier telephone cable circuit, the amplifier **300** being designed for amplifying simultaneously waves of a number of carrier telephone messages and carrier telegraph messages, extending over a frequency range from 8 kilocycles to 100 kilocycles and transmitted over one of a large number of pairs of non-loaded conductors (of 16 gauge, for example) in a lead sheathed cable. The oppositely directed path for these messages may be, for example, another such pair in another such cable. The distance over which the transmitted message is to be sent can, for example, be one or more thousand miles, and the space between repeaters (which depends on the transmitted frequency range and the desired quality of transmission) may be, for example, approximately twenty miles, the 8-kilocycle and 100-kilocycle gains of the amplifier with its input and output transformers being of the order of one-hundred db or more without feedback and being reduced by feedback to an 8-kilocycle gain of the order of 20 db or less and a 100-kilocycle gain of the order of sixty db. The 40 db difference between the 8-kilocycle and 100-kilocycle gains, with feedback, is obtained by means of attenuation equalizing or compensating means (referred to hereinafter) in the feedback path between the plate of the last tube and the grid of the first tube, and is for compensating for non-uniform attenuation of approximately twenty miles of the cable circuit over the utilized frequency range of 8 kilocycles to 100 kilocycles. With large values of $|\mu\beta|$ and power output as employed in this amplifier in the utilized frequency range, $|\mu\beta|$ and Φ should be carefully controlled to avoid danger of singing. (In practice this usually means

$$\mu\beta = |\mu\beta| \underline{|\Phi}$$

is such that $\Phi \neq 360°$ (nor to 0° nor to any integral multiple of 360°) for $|\mu\beta| \geqslant$ unity, this restriction being always sufficient, although, as indicated above, not in all cases necessary.) That is, the amplifier should be carefully designed with reference to the phase shifts and gains around the loop circuit at all frequencies within and without the utilized frequency range, and consequently with reference to the phase shifts and attenuations introduced by the interstage coupling circuits, the feedback means and the distributed capacity in the amplifying system, for instance. A number of appropriate circuit constants for this amplifier will be given hereinafter to facilitate practice of the invention. However, such values are merely illustrative, and the invention is not limited thereby.

A direct current by-pass set **306** comprising resistances **307** and condensers **308**, **309** and **310** is used to by-pass direct current around the amplifier, for example, direct current telegraph signals or direct current used for locating trouble in the cable.

The incoming circuit **304** terminates in an input transformer **315** having an impedance ratio of 130:60,000, which is shunted on its secondary side by a 70,000-ohm potentiometer resistance **316** which corrects the impedance presented to line **304** and thus reduces reflections.

The output circuit of the amplifier is connected to the outgoing or load circuit 305, which comprises amplifier output transformer 318, through an impedance bridge 320. The transformer 318 has an impedance ratio of 3500:130. The impedance bridge 320 forms an attenuation equalizer in the manner disclosed in the copending application of A. L. Stillwell, Serial No. 585,216, filed January 7, 1932 entitled "Wave translation systems", patented as No. 1,993,758, March 12, 1935. The four ratio arms of the bridge are designated by their impedance values R_0, KR_0, KR and R.

R_0 is the plate-to-cathode impedance of the space path of tube 303.

KR_0 comprises (in addition to an 83-ohm resistance 321 in series with the plate battery and a 1.6 microfarad condenser 322 in parallel to the resistance 321 and the plate battery) two impedances (in series with this parallel combination). One of these two is an inductance 323 of .00012 henry and the other is a parallel combination of a 22400-ohm resistance 324, a .01 microfarad condenser 325 and a 3-henry inductance 326.

KR is a 3,500-ohm resistance.

R is a .124-henry inductance.

The tubes 301 and 302 are Western Electric Company type 259-A tubes, which are screen grid, indirectly heated cathode tubes of high amplification factor, or so-called high mu tubes. Tube 303 may be a coplanar grid tube of the general type referred to above. This tube is operated as a power tube by employing a high value of control grid negative biasing potential and a high value of positive bias on the coplanar grid, then the control grid input voltage wave can have a large amplitude (i. e., a large grid swing). The grid 330 serves as a control grid receiving the signal voltage transmitted from tube 302 and is maintained at negative potential by a negative biasing voltage applied from a —70 volt battery 332 through a 2,000-ohm resistance 333 and a two megohm resistance 334. The grid 331 serves as a space charge grid maintained at positive potential by a positive biasing voltage applied from an 85-volt tap of plate battery 335 through a 100-ohm resistance 336. The presence of this coplanar positive grid reduces the plate resistance, R_0, to a relatively low value which enables the tube to deliver more power than if the resistance R_0 had a larger value. Tube 303, as well as tube 301 and tube 302, may have its cathode indirectly heated. The filaments of the three tubes are heated from a filament voltage supply transformer 337, which has a secondary winding 338 supplying two volts to the heater elements of tubes 301 and 302 in parallel, and has a secondary winding 339 supplying five volts to the heater element of tube 303. The cathode of tube 303 and the mid-points of windings 338 and 339 are connected to the earthed pole of the plate battery 335.

The battery 335 supplies the plate potentials and the screen grid potentials for the amplifier from a 170-volt terminal. The space current for tube 301 flows from the battery through a 250-ohm resistance 340, a network 341 of interstage coupling impedances, an inductance of 111.8 millihenries 342, tube 301 and a 600-ohm resistance 343 to the grounded pole of the battery. The network 341 comprises inductances 344 and 345, capacities 346 and 347 and resistance 348 all connected as shown, the respective values of these impedances being 4.37 henries, 19.9 millihenries, 95.5 micro-microfarad, 95.5 micro-microfarad and 25,000 ohms.

The space current for tube 302 flows through a 250-ohm resistance 350, a network 351 of interstage coupling impedances, a 123.6 millihenry inductance 352, tube 302 and a 600-ohm resistance 353 to the grounded pole of the battery 335. The network 351 comprises inductances 354 and 355, capacity 356 and resistance 358 all connected as shown, the respective values of these impedances being 1.21 henries, .04 henry, 44 micro-microfarad and 51,337 ohms.

The space current for tube 303 flows through KR_0, the primary winding of transformer 318 in parallel with KR and R in series, and the tube 303.

The potentials for the screen grids of tubes 301 and 302 are respectively supplied from the 170-volt terminal of battery 355 through a 100,000-ohm resistance 360 and a 90,000-ohm resistance 361, tubes 301 and 302, 600-ohm resistances 343, and 353 respectively.

The voltage drop across resistance 343 supplies negative biasing potential for the grid of tube 301 through a portion of a gain control potentiometer referred to hereinafter, and the secondary winding of transformer 315 and the potentiometer 316.

The voltage drop across resistance 353 supplies negative biasing voltage for the grid of tube 302 through a two megohm resistance 363. Each of the two interstage blocking condensers 364 and 365 has a capacity of 0.1 microfarad.

Across the resistances 343, 360, 340, 353, 361, 350, 333 and 336, respectively, are connected condensers 366, 367, 368, 369, 370, 371, 372, and 373, each of .75 microfarad capacity. The condensers are used to by-pass high frequency currents around the resistances to the ground bus. These condensers and resistances, and also the condenser 322 and resistance 321, suppress noise and reduce undesired feed-back and singing tendencies, and keep the transmitted frequencies out of the power leads, facilitating the use of one battery and power supply for several amplifiers.

In one diagonal of the bridge 320 is the primary winding of the output transformer 318. In the other diagonal and conjugate to the output transformer is the feedback path, the path by which part of the output is fed back to the input, containing a 1-microfarad stopping condenser 389, a supplementary attenuation equalizer 374, the gain control potentiometer 362 and a resistance 390 variable for compensating for changes, due to temperature variations in the transmission characteristics of the section of cable circuit assigned to the amplifier. The required variations of resistance 390 can be produced by any suitable means, as for example, by means responsive to temperature changes of the line section to be equalized, as indicated by legend on the drawings. The effect of the supplementary equalizer 374 is to supplement the effect of the bridge equalizer 320 as regards the equalization provided by the feedback amplifier. That is, the two equalizers together cause the amplifier to produce the desired compensation for the transmission characteristic of approximately twenty miles of the cable circuit.

The supplementary equalizer comprises inductances 375, 376, 377 and 378, capacities 379, 380, 381 and 382, and resistances 383, 384, 385, 386, 387 and 388, all connected as shown, and having respectively the impedance values of .0304 henry, .0103 henry, .204 henry, .00301 henry, .0166 microfarad, .000251 microfarad, .00249 microfarad, .00086 microfarad, 659 ohms, 659 ohms, 2,540 ohms, 2,540 ohms, 9,000 ohms and 1,150 ohms.

In the by-pass set **306**, each of the resistances **307** has a value of 500 ohms, each of the capacities **308** is 1-microfarad, the capacity **309** is 4-microfarad, and each of the capacities **310** is 1.08-microfarad.

The condensers **366** and **369** assist in giving the phase shift Φ at frequencies below the useful range a value that facilitates compliance with Nyquist's rule for freedom from instability.

Tubes **301** and **302** have an amplification factor of approximately 800, and tube **303** has an amplification factor of approximately 5.2. The plate impedances of tubes **301** and **302** are approximately 750,000 ohms and for tube **303** the plate impedance is 3,500 ohms.

The impedance elements in the interstage coupling circuits assist in making the values of $|\mu\beta|$ and Φ such as to avoid danger of singing and yield the required gain-frequency characteristic and harmonic or distortion suppression over the desired frequency range. Further, the elements assist in giving $|\mu\beta|$ and Φ the proper relationship for the desired stability.

The input and output transformers have double shields as shown, which more completely shield and separate the amplifier circuit proper from undesired circulating currents in the cables.

Over the utilized frequency range, with the large amounts of negative feedback employed, $|\mu\beta|$ is much greater than unity, and the effective gain of the amplifier from a voltage across the high impedance side of the input transformer to the fictitious driving generator in the plate circuit of the last tube is approximately equal to the loss in the feedback path, including the losses in the bridge equalizer and the supplementary equalizer. Over the utilized frequency range, $|\mu\beta|>>1$, and $|\beta|<1$. Thus $|\mu|>>1>|\beta|$. For example, at 100 kilocycles the values of $|\mu|$ and $|\beta|$, respectively, approximate 23,170 and 0.00214.

The supplementary equalizer furnishes correction so supplementing that given by the bridge equalizer that the two give the required equalization.

If it is desired to improve the distortion with feedback over and above what it was for some previous condition, it is possible to accomplish this in various ways. For example, if the μ of an amplifier is increased and no change is made in the value of β the gain of the amplifier with feedback will remain the same but the distortion will have been improved approximately the amount that μ was increased; or, if μ is kept unchanged and β is increased, the gain of the amplifier with feedback will be reduced by the increase in β and also there will be an improvement in the distortion proportional to the increase in β; and moreover there are many possible combinations of changes in β and μ which will produce changes in gain and distortion production.

An increase in μ can be accomplished for instance by substituting pentode tubes **301′** and **302′** of Fig. 65A, now to be described, for the 259-A type tubes **301** and **302** and making appropriate attendant changes in the values of the elements in the power supply and interstage circuits.

Fig. 65A is intended to represent a negative feedback amplifier which is the same as the amplifier of Fig. 65, except for the substitution and changes just mentioned. The changes are as follows: The capacities of elements **366**, **367**, **368**, **369**, **370** and **371** become .25 microfarad, .25 microfarad, 1. microfarad, 0.1 microfarad, .01 microfarad and 1. microfarad, respectively; the capacities of elements **373**, **322** and **356** become 1.5 microfarads, .75 microfarad and 60 micro-microfarads, respectively; the resistances of elements **343**, **360**, **353** and **361** become 400 ohms, 20,000 ohms, 400 ohms and 20,000 ohms, respectively; and the inductance of element **355** becomes .545 henry. The tubes **301′** and **302′** may each have an amplification factor of approximately 1,000 and a plate impedance of approximately 700,000 ohms. These two tubes may be, for example, Western Electric Company type 7592-A tubes.

A further example of an 8-kilocycle to 100-kilocycle negative feed-back amplifier embodying a form of the invention is the amplifier disclosed in the application of I. G. Wilson, Serial No. 606,875, filed of even date herewith, for Electric wave amplifying systems, Patent No. 1,948,976, issued February 27, 1934.

Fig. 66 shows a negative feed-back amplifier circuit embodying a form of the invention and suitable for amplifying frequencies of a range from 4 kilocycles to 40 kilocycles, as for example, for use in repeaters of an open wire multiplex carrier telephone system of the general type of the present commercial type C carrier telephone system described by H. A. Affel, C. S. Demarest and C. W. Green in the Bell System Technical Journal, July 1928, pages 564 to 629. In this amplifier, tubes **401**, **402** and **403** in cascade connection amplify waves received over incoming line or circuit **404** and transmit the amplified waves to outgoing line or circuit **405**. A number of appropriate circuit constants for this amplifier will be given to facilitate practice of the invention. However, such values are merely illustrative, and the invention is not limited thereby.

The incoming circuit terminates in a Western Electric type W-9384 input transformer **415** shunted on its secondary side by a 20,000-ohm resistance **416** and on its primary side by a 2,400-ohm resistance **416′**, these resistances assisting in giving the line the proper value of terminating impedance.

The output circuit of the amplifier is connected to the outgoing or load circuit **405**, which comprises a Western Electric type W-9385 output transformer **418**, through an impedance bridge **420**. The four ratio arms of the bridge are designated by their impedance values R_0, KR_0, KR and R.

R_0 is the plate-to-cathode impedance of the space path of tube **403**.

KR_0 comprises (in addition to a 1-microfarad stopping and filtering condenser **422** and the plate voltage supply circuit connected thereacross), a 139-ohm resistance **423**.

KR is a 2,227-ohm resistance **424** and a 500-micro-microfarad condenser **425** in parallel.

R is a 76,800-ohm resistance.

The tubes **401** and **402** are Western Electric Company type 259-A tubes. Tube **303** is a coplanar grid tube of the type described in connection with Fig. 57, for example, a Western Electric Company type 281-A coplanar grid tube. The control grid receives the signal voltage transmitted from tube **402** and is maintained at a negative potential by a negative biasing voltage applied from a 60-volt battery **430** through a 2,000-ohm resistance **431**, a winding **433** of a Western Electric Company type 9331 retardation coil **432** connected as an interstage coupling impedance, and 37,500-ohm resistance **434**. The space charge grid or coplanar grid is maintained at a positive potential by a positive biasing voltage applied from a 60-volt battery **432′** through two resistances 436 and 436′ of 100 ohms each.

The filament current for the three tubes is supplied by a 24-volt battery 437, through a circuit comprising a ballast lamp 438, the positive pole of the battery 437 being grounded.

A 130-volt battery 435 having its negative pole grounded and connected to the battery 437, supplies the plate potentials and the screen grid potentials for the amplifier. The space current for the tube 401 flows from the positive pole of the battery 435 through a 2,500-ohm resistance 440, a winding 443′ of a Western Electric Company type 9331 retardation coil 442 connected as an interstage coupling impedance, a 7,500-ohm resistance 444, tube 401, a 600-ohm resistance 445 and battery 437 to the grounded pole of battery 435.

The space current for tube 402 flows through a 2,500-ohm resistance 450, a winding 433′ of the coil 432, a 5,000-ohm resistance 451, tube 402, a 600-ohm resistance 453, and battery 437 to the grounded pole of battery 435.

The space current for tube 403 flows from the positive pole of battery 435 through a 5-ohm winding of an alarm relay 455, two resistances 439 and 439′, each of 83 ohms, the resistance 423, the primary winding of transformer 418 in parallel with the resistances 424 and R in series, the tube 403 and the battery 437 to the grounded pole of battery 435.

The potentials for the screen grids of tubes 401 and 402 are respectively supplied through resistances 460 and 461, each of 90,000 ohms, and 445 and 453 each of 600 ohms.

The voltage drop across resistance 445 supplies negative biasing potential for the grid of tube 401 through a portion of an adjustable gain control resistance 462 referred to hereinafter, and the secondary winding of transformer 415 and its shunting resistance 416.

The voltage drop across resistance 453 supplies negative biasing voltage for the grid of tube 402 through a winding 443 of the coil 442 and a 37,500-ohm resistance 463. The interstage stopping condenser 464 has a capacity of .25 microfarad; and the interstage stopping condenser 465 has a capacity of 1 microfarad.

Condensers 466 and 467, each of 1-microfarad capacity, are connected across the resistances 445 and 453, respectively. The cathode of tube 403 is grounded through a 2-microfarad condenser 468 which shunts battery 437. A 1-microfarad condenser 469 shunts resistance 460 and batteries 435 and 437 all in series. A 1-microfarad condenser 470 shunts resistance 461 and batteries 435 and 437 all in series. A 2-microfarad condenser 471 shunts resistances 436′, 436 and battery 432′ all in series. A 2-microfarad condenser 472 shunts resistance 436 and battery 432′ in series. A 2-microfarad condenser 473 shunts resistance 431 and batteries 432 and 437 all in series. A 1-microfarad condenser 474 shunts the resistance 440 and the batteries 435 and 437 all in series. A 1-microfarad condenser 475 shunts the resistance 450 and the batteries 435 and 437 all in series. The condenser 422 shunts batteries 437 and 435 and resistances 455, 439 and 439′ all in series; and a 1-microfarad condenser 476 shunts the batteries 437 and 435 and the resistances 455 and 439 all in series. These various condensers and resistances suppress noise and reduce undesirable feedback and singing tendencies. Condensers 466 and 467 prevent resistances 445 and 453, respectively, from decreasing the gain of the amplifier by negative feedback.

In one diagonal of the bridge 420 is the primary winding of the output transformer 418. In the other and conjugate diagonal of the bridge are the variable resistance 462 and a 1 microfarad stopping condenser 489 in series. The total resistance 462 has three sections 491, 492 and 493, of 72.5 ohms, 173.5 ohms and 714 ohms, respectively. Thus an adjustable portion of the resistance 462 is common to the output and input circuits, and the voltage across this resistance is fed back to the grid of tube 401 through the secondary winding of transformer 415 and its shunting resistance 416.

The gain of this amplifier reaches a peak of approximately 95 db without feedback and with feedback, can be adjusted to have a fixed gain of 50 db over the operating range.

Over the ultilized frequency range,

$$|\mu\beta|>>1, \text{ and } |\beta|<1. \text{ Thus } |\mu|>>1>|\beta|.$$

The condenser 425 functions to reduce the feedback voltage at frequencies above the useful range.

Fig. 67 shows gain-frequency characteristics for a three-stage high quality voice frequency (program transmission) negative feedback amplifier (not shown). The upper curve was taken without feedback; and the other three curves were taken from three different negative feedback adjustments, respectively. The amplifier was of the double bridge type shown in Fig. 5 and the adjustments for different amounts of feedback were made by adjusting a network which was connected in the feedback path between the two bridges, as is the network f of Fig. 5. Curve No. 4 was taken with the network attenuation at the value zero. In the case of this curve, the deviations from a flat characteristic are due to the input and output coils which are outside of the feedback loop. Curve No. 3 shows how the feedback corrects for these imperfections in the input and output coils, this curve being more nearly flat than curve No. 4.

Fig. 68 gives plots of actual measurements taken on a 4 kilocycles to 40 kilocycles, three-stage, negative feedback amplifier similar to the amplifier of Fig. 66, showing the improvement that the feedback effects with respect to reduction of second and third harmonics. For example, the third harmonic curve for a 25 milliampere load, shows the third harmonic to be 25 db below the fundamental for zero feedback (i. e., for zero change in gain due to feedback) and to be 85 db below the fundamental for a feedback that reduces the gain 60 db. Thus the third harmonic is 85 db—25 db or 60 db farther below the fundamental in the latter case than in the former case. This 60 db improvement is a one thousandfold improvement as regards current or voltage amplitude and is a one millionfold improvement when expressed as a power ratio. Likewise, it can be seen from the curves for second harmonics that with a feedback reducing the gain 60 db, for example, the feedback effects a 60 db improvement as regards the ratio of fundamental to second harmonic for a given power output of fundamental.

Fig. 69 is a schematic circuit diagram of a feedback system which includes input coil 503, output coil 504, μ-system of one screen grid tube 500, and feedback paths 505 and 506. The value of β is determined by varying the value of impedances 505 and 506. The larger β is made (that is the nearer to unity or the smaller the value of impedances 505 and 506) the greater will be the change in gain due to feedback. With the feedback path connected as shown and the voltage across the output circuit **502** determining the voltage fed back, the output load voltage is stabilized against variations of the load impedance and, the larger β is, the more nearly the load voltage will be independent of the load impedance. Similarly this method of feedback reduces the impedance looking into the input coil **503** which means, with large amounts of feedback the input circuit **501** is terminated by the two impedances **505** and **506** in series. Likewise the impedance which will be presented, as a result of this feedback process, to the output circuit **502** is the series combination of impedances **505** and **506**.

Inasmuch as the output coil **504** is in the μ-circuit any non-linear response or modulation products introduced by this element will be reduced by feedback, similar to the improvement in distortion introduced by the non-linear effects of the vacuum tube. The plate voltage supply source (not shown) has its positive pole connected to point **507** and its negative pole connected to point **507'**. Condenser **508** is a by-pass condenser for alternating current and a stopping condenser for the plate battery voltage. Screen grid potential is supplied through resistance **509** which is by-passed by condenser **510**. Resistance **511**, by-passed by condenser **512**, furnishes grid biasing voltage for the control grid.

Fig. 70 gives gain-frequency characteristics of this circuit. In this figure curve **521** gives the gain for each frequency from 500 cycles to 80,000 cycles without feedback. The difference in gain for different frequencies is due to the transmission through the input and output coils. However, with greater and greater amounts of feedback (curves **522**, **523** and **524** respectively) the gain becomes more nearly constant for the voice frequency range from 500 cycles to 10,000 cycles and the feedback process has improved the transmission quality of the two coils **503** and **504**.

Fig. 71 (shown on the sheet with Fig. 58) gives gain frequency characteristics of the amplifier of Fig. 65, with the supplementary equalizer omitted. The upper curve is the characteristic taken without feedback, and the lower curve is the characteristic taken with feedback through the bridge equalizer. The design of this equalizer is such as to make the gain-frequency characteristic of the amplifier approximately the same shape over the utilized frequency range, as the loss-frequency characteristic of the cable section assigned to this amplifier for equalization.

Fig. 72 discloses a two-stage amplifier operating between an incoming circuit **525** and an outgoing circuit **526** and comprising vacuum tubes **527** and **528**. Tube **527** is coupled to the incoming circuit by input transformer **534**, shunt resistances **536** and **535** being used to provide the proper terminations for incoming line **525**. Tube **528** works into outgoing circuit **526** through step-down transformer **529**. On the output side of output transformer **529** is the resistance bridge comprising arms R, KR and KR_0, the fourth arm of this bridge being the resistance marked R_0 on the figure, this being the resistance looking back into the output transformer.

Tube **527** is a heater type tube and has a resistance **532**, in one example 1800 ohms, connected in the grid circuit and also in the direct current plate circuit so that this resistance is common to both the grid and the plate circuit. The steady drop of potential across resistance **532** aids in biasing the grid of tube **527** negatively and a battery **533** may also be used to provide additional grid bias.

The heating current for the filaments of tubes **527** and **528** is supplied from battery **537** through regulating resistance **538**. Space current for both tubes is supplied from plate battery **543**. Screen grid voltage for tube **527** is obtained from battery **539**. A positive potential is applied to one of the coplanar grids of tube **528** from battery **540**. The coupling circuit between tubes **527** and **528** comprises series capacity **545** and shunt resistances **542** and **544**. Battery **541** applies negative grid bias to one of the coplanar grids of tube **528**.

A feedback connection is provided by conductor **531** from a point on the output bridge conjugate to the output circuit **526**, this conductor leading back to the upper side of resistance **532** in the drawings so that resistance **532** is included in the feedback path.

In this circuit the closed path includes the tubes **527** and **528** and also the output transformer **529**, together with the feedback circuit above traced. Since the closed path includes the output transformer **529** this transformer aids in giving the necessary phase shift around the entire closed path. A further advantage of including the output transformer **529** in the closed path is that the operation of the feedback process reduces the effect of the non-linear transmission characteristic of the transformer.

A circuit of the type disclosed in Fig. 72 has been successfully used in high quality voice frequency program transmission where it was necessary to transmit with minimum distortion a wide frequency band extending from about 35 cycles to the neighborhood of 10 kilocycles.

Fig. 73 shows an amplifier which illustrates repetition of the feedback process. The inside closed loop consists of an input bridge **551**, μ-system **550** which may be a vacuum tube, and output bridge **556**. The loss through the β-circuit of the inside loop is to be a constant over the range of useful frequencies and stabilize the μ-system against variations in the amplification of the system **550** for the second feedback process. The system in which the second feedback process takes place consists of an input bridge **561**, a μ-system which is the inside closed loop just described, output bridge **562**, and networks **571**, **572** and **573** in the β-circuit **563** for the purpose of producing the proper gain-frequency characteristic, varying the shape of this characteristic as may be desired for example to take care of changes in cable attenuations with temperature, and correcting for the phase distortion introduced by the preceding cable. The feedback path from bridge **556** to bridge **551** is through conductors **557** and **558**, and this path is rendered conjugate to output transformer **560** and input transformer **559** by these two bridges. The bridge **556** comprises ratio arms R_0, KR_0, KR and R as in Fig. 5, for example. The bridge **551** has ratio arms designated **552**, **553**, **554** and **555**. The bridges **561** and **562** render the path through networks **571**, **572** and **573** conjugate to the incoming and outgoing circuits **565** and **564**. The ratio arms of the bridge **562** are designated R_{02}, $K'R_{02}$, $K'R_2$ and R_2; and the ratio arms of bridge **561** are the primary windings of transformer **559** and arms **566**, **567** and **568**.

Above, in this specification, under the heading II. c. Stability of gain with respect to variations in $|\mu|$, it was pointed out that if the secant of Φ equaled $|\mu\beta|$ then the amplitude of the transmitted wave would be stabilized against variations in the absolute magnitude of μ. The calculated variations of the gain, expressed in decibels, with feedback is plotted in Fig. 10 as a function of variation in gain without feedback due to a variation in $|\mu|$. Curves marked +1 db, +2 db, +3 db, +5 db and +8 db are examples of cases of positive feedback in which the gain with feedback does not vary as much as the gain without feedback for small variations. Fig. 11 illustrates this fact for small variation also, inasmuch as it is only an enlargement of Fig. 10 about the point 0,0 for values of positive feedback which produce 8, 10, 13, 20 and 30 db increase in gain.

Fig. 74 is a generalized schematic illustrating application of the above principle. This circuit consists of an input circuit **577** and an output circuit **578** connected to an input coil **579** and an output coil **580**. The input coil has two secondary windings **581** and **582**; and the output coil has two primary windings **583** and **584**. The amplifying device connected between the input and output coils **579** and **580** is made up of two similar amplifiers, connected together through a special feedback circuit **590** to be described later. The input signal is applied to both bridges **585** and **587**, windings **581** and **582** are poled so as to make the grid of amplifying device **575** opposite in sign to the grid of amplifying device **576**. Amplifying devices **575** and **576**, while they must be similar, may consist of one, two or more tubes. The plate resistance R_0 of the last tube is to constitute one arm of the output bridge **586** and **588**, respectively. Connected across one diagonal of these two output bridges are the two primary windings of the output coil **583** and **584**. The other two diagonals of the bridge, which are conjugate to the output diagonals, contain the feedback path **589**. The elements KR_0, KR, and R are impedances and obey the laws set down throughout the above discussion, for example as referred to in connection with Fig. 5.

Conductors **589** should be considered as two separate feedback paths, one from the output of amplifying device **575**, the other from the output of amplifying device **576**. These two feedbacks enter network **590** which should introduce sufficient phase shift and attenuation holding $|\mu\beta|=\sec\Phi$; further network **590** should direct the feedback from amplifying device **575** to bridge **587** if the number of tubes in **575** is odd or direct the feedback from **575** back to bridge **585** if there are an even number of tubes in **575**, likewise **590** should connect the feedback from bridge **588** to **585** if **576** has an odd number of phase reversals or connect **588** to **587** if an even number of phase reversals are produced in amplifying device **576**. This system may then be utilized to increase the gain of a given device and further reduce the variations in the over-all gain due to variations in $|\mu|$.

Fig. 75 is a specific example in which the gain of the space discharge device **600** (in this case a heater type vacuum tube) is increased approximately 9 db and if the gain changes ±.2 db without feedback the change with feedback is only about −.02 db and −.05 db. The circuit consists of an input and output circuit **601** and **602** connected to input and output coils **603** and **604** which are respectively in diagonals of input and output bridges **605** and **606**. Connected between the two diagonals conjugate to the input and output coils, is the feedback path **607**. The feedback signal leaves the output bridge along conductors **607** and goes through a lattice network **608** which introduces 180° phase shift and presents an impedance of 1000ω at terminals **621** and **622**. The fedback signal continues through network **609** which is a two-section phase shifter, each section consisting of a T-network. The first T-network has a shunt arm consisting of a resistance **617** with a value of 973ω and a condenser in series with this resistance **619** with a value of .0014 mf. The series arm of the T-network consists of two equal resistances **614**, and **615** presenting 412 ohms resistances, and then across the two series arms is shunted an inductance **611** with 1.44 m. h. The second section is very similar to the first except in that the capacity **618** in the shunt arm is .0158 mf. instead of .00144 mf. and the shunting inductance around the two series arms **610** is 15.8 m. h. instead of 1.44. The resulting effect of this network upon the fedback signals as they pass through is to shift their phase by 20°±1/4°. With $|\mu\beta|=1.066$ for the closed loop the signal will be stabilized with feedback and increased in amplitude with feedback. Since the cosine of a 20° angle is 0.9397 it is seen that this is equal to the reciprocal of $|\mu\beta|$ as here given and that this amplifier with the constants given above is operating on boundary **C**.

Fig. 75A is a network which can be used in the feedback path instead of network **609**. This other network **609'** is very well suited for a range of frequencies extending from 30 kilocycles to 40 kilocycles.

In circuits using input or output bridges, it is possible to vary the $\mu\beta$ of the system without affecting the input or output impedances, and likewise it is possible to control these impedances without changing $\mu\beta$. Moreover, in any given case, it is possible to control the input or output impedance and also to control $\mu\beta$ to make it any required value. For example, the output or the input bridge may be unbalanced to any extent to control the influence of feedback upon the impedance as seen from the output or input respectively. This will tend to alter the value of $\mu\beta$. However, an adjustment may be made at any suitable point in the μ-circuit or in the β-circuit to restore the value of $\mu\beta$ to the value it had before the assumed change in the bridge took place. Amplifiers or systems in which negative feedback is used to match input or output impedance to the connected impedance, and negative feedback amplifiers or systems with input or output bridges unbalanced, for example for controlling the amplifier input or output impedance, are claimed in my copending application Serial No. 663,317, filed March 29, 1933, entitled Wave translation systems.

The invention is capable of wide variation from the forms illustrated and described, its scope being defined in the appended claims.

What is claimed is:

1. In a wave translating device or system having amplifying properties, an input portion and an output portion, means to apply fundamental waves to said input portion, said system carrying fundamental components in said output portion and having means producing other wave components in said output portion, and means controlling the relative magnitudes of said components in said output portion comprising means to feed waves from said output portion to said input portion to decrease the gain of the system.

2. In a wave translating system having an input portion and an output portion, means to apply waves of given frequency to said input por-

tion, said system being subject to variable unilateral operation, and means to stabilize said system comprising means deriving from an output portion thereof waves of the same frequency as said applied waves and which tend to vary in correspondence with the variable operation of said system and means for producing negative feedback of said derived waves to an input portion in a manner to counteract such variable operation.

3. In a wave translating system operating to amplify applied fundamental waves, and to produce distortion components as a function of nonlinearity in the system, means to increase the ratio of the amplified fundamental wave component to distortion components comprising means to utilize a portion of the waves translated by said system to reduce the gain of the system below the gain with zero feedback in the system of the waves translated by the system.

4. A wave amplifying system for waves of fundamental frequency, said system having inherent distortion, and means to increase the ratio of fundamental to distortion components comprising means to utilize fundamental components present in said system to reduce the amplification of said system for said waves below the value with zero feedback of said fundamental components in said system.

5. A wave translating system comprising input and output portions, means to impress waves of a band of frequencies on said input portion, said system in the non-oscillating condition tending to produce output variations that bear other than a constant linear relation to the waves impressed on the input portion, and means to reduce said output variations comprising means to apply waves of the same band of frequencies to said input portion under control of said output portion, said impressed and applied waves combining vectorially to produce in said input portion resultant waves the ratio of whose amplitude to the amplitude of said applied waves is less than unity.

6. A wave translating system having an input, means to impress a wave on said input, and means to stabilize said system against variations in phase shift produced in waves traversing the system comprising means to return a portion of said wave to said input after it has traversed the system, said impressed and returned waves producing resultant input waves, and the ratio of amplitudes of the returned waves to the resultant waves being greater than unity.

7. A wave translating system according to claim 6 in which the cosine of the phase angle between said returned and resultant waves is substantially inversely proportional to said ratio. (Boundary C.)

8. A wave translating system according to claim 6 in which said ratio is many times larger than unity.

9. A wave amplifying system having an input and an output, means to impress a wave on said input, and means to stabilize said system against variations produced in the system in amplification ratio between output and input comprising means to feed back to the input a portion of the output wave resulting from amplification of said impressed wave, said impressed and fedback waves producing resultant input waves, and the ratio of amplitude of the fedback wave to the resultant waves being greater than unity.

10. A wave amplifying system having an input and an output, means to impress a wave band on said input, and means to reduce the gain of said amplifier for said band below the gain without feedback, comprising means feeding back to the input a portion of the output wave resulting from amplification of said impressed wave, said impressed and fedback waves producing resultant input waves, and the ratio of amplitude of the wave so fed back to the amplitude of the resultant wave being substantially greater than unity.

11. A wave translating system having an input and an output, means to impress waves of a band of frequencies on the input thereof, and means to control the amplitude-frequency relations of the resulting output waves comprising means to feed back to the input a portion of the output waves, producing thereby resultant input waves, the ratio of the amplitude of the waves so fed back to the amplitude of the resultant input waves being greater than one and said fedback waves and resultant input waves having other than phase coincidence, and means in the feeding back means for controlling the relative amplitudes of wave components of different frequencies to modify the amplitude-frequency relations of the final output waves.

12. A wave translating system having an input and an output, means to impress fundamental waves on said input, said system producing in its output said fundamental waves accompanied by distortion components, and means to increase the ratio of fundamental to distortion components in the output comprising means to feed back to the input a portion of the output waves in such amplitude and phase as to reduce the gain of said system below the gain with no feedback.

13. A wave translating system having an input and an output, a forward portion transmitting from said input to said output, a feedback transmitting from said output to said input, means to impress waves on said input, and means to control the phase rotation of waves in said forward portion comprising means to control the amplitude and phase of the waves fed back to said input, the waves so fed back reducing the gain of said forward portion.

14. In a wave translating system producing amplification in the translated waves, having an input and an output, means to control phase distortion in said system comprising means to apply to said input, waves derived from said output in such phase and amplitude as to reduce the gain of said system between said input and said output.

15. A wave translating system having a forward or μ-path and a feedback or β-path, in which $|\mu\beta|$ equals ½ secant of the angle of phase shift produced in waves traversing once around the forward and the feedback paths in tandem.

16. A wave translating system having a forward or μ-path and a feedback or β-path, in which $|\mu\beta|$ exceeds unity and equals the secant of the angle of phase shift produced in waves traversing once around the forward and the feedback paths in tandem.

17. A wave translating system having an input and an output, means to impress waves on said input, and means to stabilize said system against variable operation in response to variables in the system comprising means to feed back to the input a portion of the output waves, in which the value of $|\mu\beta|$ lies between the limits defined by the relations

$$\Phi = \cos^{-1} |\mu\beta|, \text{ (boundary B)}$$

and

$$\Phi = \cos^{-1} \frac{2+|\mu\beta|^2}{3|\mu\beta|}, \text{ (boundary H)}$$

18. The method of increasing the ratio of signal to distortion in a wave translating system in which distortion takes place, comprising impressing upon the system signal input independent of translation in the system impressing output waves from said system representing both the signal and the distortion upon the input of said system to reduce the transmission efficiency of the system and simultaneously increasing said signal input independent of translation in the system.

19. A wave translation system having an input and an output and translating waves from said input into said output, a feedback path from one point to another in said system whereby a closed wave propagation path is provided, the wave propagation constant around said closed path having a modulus substantially greater than unity for the frequencies of said translated waves.

20. A wave amplifying system having input and output and a feedback path from said output to said input whereby a closed wave propagation path is provided including an amplifier, the wave propagation constant for said closed path having a modulus substantially greater than unity for the frequencies of waves to be amplified by the system.

21. A wave translating system for waves of fundamental frequency, said system producing distortion, said system being subject to variable wave translating operation in response to variables in the system, and means comprising a feedback path for deriving wave components including distortion wave components from said system and applying the derived components to said system for both modifying the amount of distortion and rendering the system less subject to variable operation with respect to translation of a fundamental wave of a given frequency, with such feedback as compared to the system with zero feedback.

22. A signal wave translating system having an input and an output, means to impress signal waves on said input, said system having a distorting characteristic whereby distortion components appear in the output, each of said distortion components having a frequency different from that of any fundamental component producing that distortion component, and means to utilize a portion of the output waves to oppose the production in said system of said distortion components, whereby for a given output level of signal the ratio of signal level to distortion level is improved by substantially the amount of the reduction of said distortion.

23. A signal wave amplifying system having an input and an output, means to impress signal waves on said input, said system being subject to variable amplifying operation in response to variables in the system, and means to utilize a portion of the output waves to stabilize said system against such variable amplification by causing said portion of the output waves to reduce the gain of said system, whereby the improvement in stability is substantially equal to the reduction in gain.

24. An electrical wave transmission system having an input and an output, means to apply waves to said input, a load coupled to said output having an impedance varying with frequency, a feedback circuit for deriving from said system waves whose current or voltage is a function of the current or voltage input to said load and impressing them upon said input, said impressed waves and said applied waves producing resultant waves in said input, the ratio of the amplitude of said impressed to said resultant waves being large compared to unity, and the frequency amplitude relations of the said impressed waves bearing such relation to the variation of the load impedance with frequency as to maintain the current of voltage input to the load substantially constant.

25. A wave translation system, subject to variable operation, having input and output, means to apply waves to said input, a load coupled to said output, and means for maintaining the load voltage independent of variable operation of the system comprising means to derive from said system a portion of the wave that is applied to said load and to supply said derived wave to the input to produce with the applied waves resultant waves, the ratio of amplitudes of the waves so supplied to the resultant waves being greater than unity.

26. A system according to claim 25 in which said derived voltage is in shunt to the load.

27. A system according to claim 25 in which said derived voltage is in series with the load.

28. The method of obtaining an impedance representing a required function of frequency across a pair of terminals, which method comprises including said pair of terminals in a closed wave propagation path, causing the wave propagation constant around said closed path to have a modulus greater than unity and proportioning the value of said modulus and the phase angle (for once traversing said closed path) to produce the required impedance.

29. A wave amplifying system having an input and an output, a load having impedance coupled to said output, a path feeding waves from said output to said input to control the gain of said system between said input and said output, said feeding back path being conjugate to the load, whereby the gain may be controlled without affecting the impedance of the output as seen from the load.

30. A wave amplifying system having input and output, a path feeding waves from said output back to said input to reduce the gain of said system between said input and said output below the gain without feedback, a wave source having impedance coupled to said input, said feeding-back path being conjugate to said wave source whereby the gain of said system may be controlled without affecting the impedance of the input as seen from said source.

31. An electrical wave translating system having input and output, means to apply fundamental waves to said input, said system having a distorting characteristic whereby distortion products appear in said output, a feedback circuit from said output to said input, selective means in said feedback circuit for suppressing the distortion products and passing waves of fundamental frequency, the waves of fundamental frequency fed back to the input and the applied fundamental waves producing resultant input waves, the ratio of amplitude of the waves so fed back to said resultant waves being greater than unity and said last-mentioned two waves having a phase angle whose cosine is substantially inversely proportional to said ratio.

32. A system according to claim 5 in which the means to impress the waves on the input, and the means to apply the waves of the same fre-

quency to the input, are effectively in series relation whereby the input impedance to the system is high.

33. A system according to claim 5 in which the means to impress the waves on the input, and the means to apply the waves of the same frequency to the input, are effectively in shunt relation whereby the input impedance to the system is low.

34. In combination, electric space discharge apparatus comprising an anode, a cathode and a discharge control element, a work circuit for said apparatus, an output circuit for said apparatus, including said work circuit, and means for feeding from said output circuit to said control element a potential independent of the impedance of said work circuit.

35. In combination, electric space discharge apparatus comprising an anode, a cathode and a discharge control element, means for supplying to said apparatus waves causing modulation therein, an output circuit for said apparatus, having a point at a potential directly proportional to and in phase with the driving voltage in the anode-cathode discharge space of said apparatus, and means for reducing the amount of modulation in said apparatus comprising a wave transmission path for delivering said potential from said point to said control element, said potential reducing the gain of said apparatus below its value with zero feedback.

36. In combination, wave translating apparatus, means for supplying to said apparatus waves producing modulation in said apparatus, and means, capable of transmitting waves lying within the frequency spectrum of the supplied waves, for feeding wave products including odd order products of modulation from the output side to the input side of said apparatus in such phase as to reduce the magnitude of the modulation products appearing at the output side of said apparatus below their magnitude without feedback.

37. In combination, wave translating apparatus that reverses the phase of waves transmitted therethrough, means for supplying to the input side of said apparatus waves that produce modulation in said apparatus, and means capable of transmitting without phase change waves of all frequencies of a wide frequency range, for feeding wave products including odd order products of modulation from the output side to the input side of said apparatus in the phase in which they are originally generated whereby the magnitude of the modulation produced is reduced below its value without feedback.

38. In combination, wave repeating and generating means, an input path for supplying waves to said means, a wave output path for said means, and coupling means, capable of transmitting without phase change waves of all frequencies of a wide frequency range, so coupling said output path with said input path as to prevent the supplied waves from affecting the potential difference between two points on said output path except through the repeating and generating actions of said means, but to feed wave products including those representing odd order modulation products generated by said means to said input path in such phase as to reduce their intensity in said output path.

39. In combination, wave repeating and generating means, an input path for supplying waves to said means, a wave output path for said means, coupling means, capable of transmitting waves of frequencies lying within the frequency spectrum of the supplied waves, so coupling said output path with said input path as to prevent the supplied waves from affecting the potential difference between two points on said output path except through the repeating and generating actions of said means, but to feed wave products including those representing odd order modulation products generated by said means to said input path in such phase as to reduce their intensity in said output path, a work circuit, and means for supplying to said work circuit a voltage proportional to said potential difference.

40. In combination, electric space discharge apparatus comprising an anode, a cathode, a space discharge path, and a discharge control element, two circuits, a coupling circuit coupling said discharge path to said two circuits to enable it to impress electromotive forces thereon, said coupling circuit being so associated with said two circuits that said two circuits are free from effective coupling, whereby neither of said two circuits is affected by the other, and means connecting said control element to a point on one of said two circuits.

41. Electric space discharge apparatus having an anode, a cathode and a discharge control element, an external output circuit for said apparatus, a path in said output circuit, having its impedance substantially pure resistance connecting said anode and said cathode, a work circuit for said apparatus, connected across a portion of said path, and a connection from said control element to a point on said portion, electrically remote from each end of said portion.

42. In combination, electric space discharge apparatus comprising an anode, a cathode, a space discharge path, and a discharge control element, two circuits, a resistance coupling said discharge path to said two circuits to enable it to impress electromotive forces thereon, said resistance being so associated with said two circuits that said two circuits are free from effective coupling, whereby neither of said two circuits is affected by the other, and means connecting said control element to a point on one of said two circuits.

43. A Wheatstone bridge circuit having ratio arms and diagonals, electric space discharge apparatus connected in said circuit, having an anode, a cathode and a discharge control element, a work circuit effectively connected across two opposite corners of said bridge, a current path effectively connected across the other corners of said bridge, a wave transmission path between said cathode and said control element, included in said current path, the space discharge path between said anode and said cathode in said apparatus being so connected in said bridge circuit that the driving voltage in said space discharge path transmits waves to said work circuit and to said wave transmission path.

44. A network comprising two impedance branches connected in parallel, each branch consisting of two resistance elements, the ratio of the two resistances in one branch being equal to the ratio of the similarly positioned resistances in the other branch, a circuit connected between the terminals of the impedance branches, a second circuit connected between the junction points of the resistance elements in the respective branches, and electric space discharge apparatus having an anode, a cathode and a discharge control element, with the space discharge path between said anode and said cathode included in

one of said resistance elements and with the control element connected to a point on one of said circuits.

45. In combination, two circuits, means including an electric space discharge path, connecting said circuits in conjugate relation to each other and transmitting waves from said discharge path through one of said circuits, control means for controlling space discharge in said path, and means connecting said control means to a point on said one circuit.

46. A Wheatstone bridge network comprising two circuits, means including an electric space discharge path, connecting said circuits in conjugate relation to each other and transmitting waves from said discharge path through one of said circuits, control means for controlling space discharge in said path, means connecting said control means to a point on said one circuit, and means for supplying electrical variations to said control means.

47. A Wheatstone bridge circuit having external arms and a diagonal, an electric space discharge device, a space discharge path for said device, included in one of said arms, for transmitting waves from said discharge path through said diagonal, and an input circuit for said device, included in said diagonal.

48. A Wheatstone bridge circuit having ratio arms and diagonals, an electric space discharge device, a space discharge path in said device, included in one of said arms for transmitting waves from said discharge path through each of said diagonals, a work circuit for said device, included in one of said diagonals, and an input circuit for said device, external to said device and included in said other diagonal.

49. The method of increasing the output load capacity of electric space discharge apparatus which comprises negatively regenerating waves representing even and odd order modulation products in the apparatus.

50. A repeater having input and output circuits, means for deriving from the output circuit waves representing odd order modulation components originating in the repeater and not present in the input waves, and means to utilize said derived components to so control the action of said repeater as to reduce said components.

51. A repeater for amplifying simultaneously the different frequency components of a signal wave comprising a space discharge tube amplifying system having input and output circuits, and means for reducing distortion in said repeater comprising an output-to-input coupling more effectively degenerative for odd and even order distortion products than for the signal components.

52. A repeater for a multiplex carrier system comprising a space discharge tube amplifying system having input and output circuits, means to impress on the input circuit a plurality of message modulated carrier waves for simultaneous amplification, and means to prevent interference between said message modulated waves comprising means for deriving from said output circuit a complex wave differing in form from the complex wave constituted by the waves to be repeated and including odd order distortion components produced in said repeater and means responsive to said derived wave to limit the effectiveness of said repeater in producing said difference.

53. A repeater for waves that transmit signals having input and output circuits, means for deriving from the output circuit wave components including products of both odd and even order modulation originating in the repeater and foreign to the signals and not present in the input waves, and means to utilize said derived components including said odd and even order modulation products to so control the action of said repeater as to reduce said components for a given output level of signals below the value they would have with zero feedback action in said repeater.

54. The method of controlling distortion waves produced by energy amplifying means in fundamental waves passing from one portion of said means to another portion of said means, which comprises so transmitting the fundamental waves and the distortion waves from said other portion of said means to said one portion of said means as to reduce the energy amplification of said means.

55. The method of reducing the ratio of distortion to useful power in the output of energy amplifying means, which comprises so introducing a sufficient part of the fundamental output energy of said means and a sufficient part of the distortion output into a portion of said means anterior to the output portion of said means as to reduce the energy amplification of said means proportionally to the distortion reduction and sufficiently to produce the required amount of distortion reduction.

56. In combination, a transmission circuit having a gain, means for reducing distortion originating in said circuit from its value for merely forward transmission in the circuit, said means comprising a feedback path feeding back in said circuit from the output side of said circuit waves which reduce said gain from its value for merely forward transmission in the circuit, and transmission control means in said path.

57. A wave translation system comprising a circuit having a gain, a transmission control network having a given effect upon transmission passing through said network, and means for reducing distortion originating in said circuit from its value for merely forward transmission in the circuit, said means comprising a feedback path connecting said network to feed from the output side to the input side of said circuit waves which reduce said gain from its value for merely forward transmission in the circuit and to produce an effect inverse to said given effect upon transmission passing through said system.

58. A regenerative vacuum tube circuit comprising a path for feeding back waves from the output side to the input side of said circuit in such phase and amplitude as to reduce the gain of the circuit and the distortion originating in said circuit below their values for merely forward transmission in the circuit, a network having a given transmission characteristic and means connecting said network in said path to contribute to said circuit a transmission characteristic the inverse of said given characteristic.

59. In combination, a transmission circuit, a feedback path from the output side of said circuit to an anterior portion of said circuit, and a transmission control circuit connected in said path for producing in said path attenuation and phase shift varying with frequencies in the same manner as the attenuation and phase shift of said transmission circuit.

60. In combination, a wave transmission circuit, an electric space discharge tube amplifying system associated therewith comprising an amplifying space discharge device, a path for pro-

ducing feedback action in said system in such amplitude and phase as to reduce modulation originating in said amplifying space discharge device, and a gain regulating transmission equalizing network in said path, having its attenuation-frequency characteristic similar to that of said circuit.

61. In combination, a wave transmission circuit, an electric space discharge tube amplifying system associated therewith comprising an amplifying space discharge device, a path for producing feedback action in said system in such amplitude and phase as to reduce non-linear distortion originating in said amplifying space discharge device, and gain controlling and transmission equalizing networks in said path, having their attenuation-frequency characteristic similar to that of said circuit.

62. A system comprising a transmission circuit, an incoming circuit for applying waves to said transmission circuit, an outgoing circuit for said transmission circuit, a feedback path for feeding waves from the output of said transmission circuit to the input of said transmission circuit, the amplitude ratio of the fed back waves to the resultant of the fed back waves and the applied waves being greater than unity, and differentially acting means for preventing feedback action in said system from affecting the impedance into which said incoming circuit works.

63. A system comprising a transmission circuit, an incoming circuit for applying waves to said transmission circuit, an outgoing circuit for said transmission circuit, a feedback path for feeding waves from the output of said transmission circuit to the input of said transmission circuit, the amplitude ratio of the fed back waves to the resultant of the fed back waves and the applied waves being greater than unity, and balancing means for preventing variations in the impedance of said incoming circuit from affecting feedback action in said system.

64. A system comprising a transmission circuit, an incoming circuit for applying waves to said transmission circuit, an outgoing circuit for said transmission circuit, a feedback path for feeding waves from the output of said transmission circuit to the input of said transmission circuit, the amplitude ratio of the fed back waves to the resultant of the fed back waves and the applied waves being greater than unity, and means rendering said input circuit and said feedback path conjugate to each other.

65. A system comprising an amplifier, an incoming circuit for said amplifier, an outgoing circuit for said amplifier, a feedback path for said amplifier, and balancing means for preventing feedback action in said system from affecting the impedances with which said amplifier faces said circuits and preventing variations in the impedances of said circuits from affecting the feedback action of said amplifier.

66. A system comprising an amplifier, an incoming circuit for said amplifier, an outgoing circuit for said amplifier, a feedback path for said amplifier, and means for preventing feedback action in said system from affecting the impedances with which said amplifier faces said circuit and preventing variations in the impedances of said circuits from affecting the feedback action of said amplifier, said means comprising a Wheatstone bridge connecting said incoming circuit and said feedback path in conjugate relation to each other and a Wheatstone bridge connecting said outgoing circuit and said feedback path in conjugate relation to each other.

67. A system comprising an amplifier, an incoming circuit for applying waves to said amplifier, an outgoing circuit for said amplifier, a feedback path for feeding waves from the output of said amplifier to the input of said amplifier in such amplitude and phase as to reduce non-linear distortion originating in said amplifier, the amplitude ratio of the fed back waves to the resultant of the fed back waves and the applied waves being greater than unity, and means for preventing feedback action in said system from affecting the impedance into which said incoming circuit works.

68. A system comprising an amplifier, an incoming circuit for said amplifier, an outgoing circuit for said amplifier, a feedback path for said amplifier for feeding back waves in such phase as to reduce distortion originating in said amplifier, and means for preventing variations which occur in the impedance of said incoming circuit from affecting feedback action in said circuit, and means connecting said outgoing circuit and said feedback path in conjugate relation to each other.

69. A system comprising an amplifier for a multiplex carrier telephone cable circuit, an incoming circuit for connecting said cable circuit to said amplifier, an outgoing circuit for connecting said amplifier to said cable circuit, a feedback path for said amplifier for feeding back waves of a wide frequency range in such phase as to reduce distortion originating in said amplifier, gain controlling and transmission equalizing means for compensating for the variation of attenuation of said cable with temperature changes of said cable, and means connecting said gain control and transmission equalizing means in said feedback path for reducing the deleterious effects of the resistance noise from the gain control and transmission equalizing means and the deleterious effects of tube noise generated in the amplifier upon the signal-to-noise ratio in said outgoing circuit.

70. A signaling system comprising a cable circuit and an amplifier in said circuit having gain-reducing feedback means for compensating for effects produced by the transmission characteristics of said circuit upon transmission passing through said circuit.

71. A wave transmission system comprising a circuit for transmitting waves of a wide frequency range, said circuit being subject to greater disturbance for one frequency range than for another and having a greater attenuation for said other frequency range than for said one frequency range, an amplifier associated with said circuit for amplifying said waves, said amplifier comprising a feedback path producing negative feedback in said amplifier, and an attenuation equalizing means in said feedback path having an attenuation frequency characteristic similar to that of said circuit.

72. A signaling system comprising a carrier telephone cable circuit for transmitting waves of a large number of carrier telephone channels of different frequency ranges, said circuit being subject to greater disturbance for the low frequency channels than for the high frequency channels, an amplifier connected in said circuit for amplifying said waves, said amplifier comprising means for feeding back waves in such phase as to reduce distortion produced in said amplifier, and means included in said feedback means having an attenuation characteristic similar to that of said circuit.

73. A carrier signaling system comprising an amplifier, a carrier wave transmission circuit with a flat noise level spectrum for transmitting to said amplifier a plurality of carrier signal channels of different frequency ranges with substantially equal side band levels at the output of the amplifier, and means in said amplifier for causing the amplification of the amplifier to be greater for the high frequencies than for the low frequencies of the transmission range and tending to cause distortion produced in the amplifier to be less for low frequencies than for high frequencies.

74. A carrier wave signaling system comprising an amplifier, a carrier telephone cable circuit so shielded that substantially the only noise in the circuit is thermal noise, for transmitting to said amplifier waves of a plurality of carrier telephone channels of different frequency ranges with substantially equal side band levels at the input of the amplifier, said channels including a frequency band extending substantially from 4 kilocycles to 40 kilocycles, a feedback path for said amplifier for feeding back waves in such phase as to reduce the gain and the distortion produced in the amplifier, and an attenuation equalizer in said path having attenuation increasing with frequency for producing greater amplifier gain reduction and distortion reduction at low frequencies than at high frequencies.

75. An electrical wave transmission system having an input and an output, means to impress waves on said input, an outgoing circuit, a transformer coupling said outgoing circuit to said output, a feedback path from a point in said outgoing circuit following said transformer to said input, said feedback path applying waves to said input having the same frequency as said impressed waves to form with said impressed waves resultant input waves, and the ratio of amplitude of the fedback to resultant input waves being greater than unity.

76. An electrical wave transmission system having an input and an output, an incoming circuit carrying waves to be impressed on said system, a transformer coupling said incoming circuit to said input circuit, a feedback path from the output to a point in the incoming circuit ahead of said transformer, said feedback applying waves to said input of the same frequency as said impressed waves to produce resultant input waves, and the ratio of amplitude of the fedback to the resultant waves being greater than unity.

77. A multi-stage space discharge tube circuit, each stage comprising a space discharge device having an input and an output, and feedback circuits individual to said stages, each such feedback circuit having a phase shift with frequency over the transmitted band and reducing the gain of the individual stages non-uniformly over the transmitted band to control the transmission and phase characteristic of the multi-stage circuit as a whole.

78. A wave amplifying system according to claim 10 in which said means feeding back to the input a portion of the output wave comprises means for amplifying said wave portion.

79. A wave translation system according to claim 19 in which an input is coupled to said closed path at one point and an output is coupled to said closed path at another point, said closed path including wave amplifying means between said two points and amplifying waves in the direction from said input to said output.

80. A wave translation system according to claim 19 in which an input is coupled to said closed path at one point and an output is coupled to said closed path at another point, said closed path including wave amplifying means between said two points and amplifying waves in the direction from said output to said input.

81. A multiplex carrier wave amplifier for simultaneously amplifying waves representing a plurality of voice-modulated carrier waves covering a range of the order of 40 kilocycles, said amplifier having a feedback path for feeding back from the output to the input the total frequency range being amplified in such manner as to reduce the gain of the amplifier for waves of said range and increasing the stability by a factor comparable to the factor of gain reduction.

82. An amplifier according to claim 81 in which said feedback path includes a wave-shaping network for altering the frequency-amplitude relations of the waves within said frequency range.

83. A wave amplifying system having an input and an output, a plurality of separate feedback paths leading from said output to said input, and separate wave transmission devices in said feedback paths for separately controlling transmission through said feedback paths, the feedback through certain of said paths being such as to reduce the gain between said input and said output.

84. A wave amplifying repeater for insertion into a signal line, said repeater having a forward portion giving a gain ratio of several times ten decibels, and a feedback portion reducing the gain of said forward portion and stabilizing said repeater against changes in gain in said forward portion to the order of hundredths of 1 db.

85. A wave amplifying repeater according to claim 84 in which the said feedback portion reduces the gain of said forward portion and stabilizes said repeater against changes in gain in said forward portion to the order of thousandths of 1 db.

86. An electrical wave transmission system comprising a line with amplifying repeaters inserted therein at intervals, said line including such repeaters to the number of the order of one hundred in tandem, each such repeater having a feedback path from output to input reducing the gain of said repeater and stabilizing said repeater against variations in gain with circuit variables, such that the variation in gain of the entire circuit from end to end is of the order of less than ten db.

87. A system according to claim 86 in which the variation in over-all gain is of the order of one db.

88. A wave amplifying system according to claim 10 in which the amplitude of the amplified wave appearing in the output of the system is several times 10 db greater than said resultant input amplitude.

89. A wave translation system comprising a μ-circuit and a β-circuit in which $|\mu|$ is many times larger than $|\beta|$.

90. A wave translation system comprising a μ-circuit and a β-circuit, in which the value $|\mu\beta|$ is many times larger than one.

91. The method of maintaining more nearly constant the relation between gain and load of a space discharge system with varying load, which comprises feeding back waves in said apparatus to reduce the gain of said system.

92. A wave translation system having an input and an output, means to apply waves to said

input to be amplified, said system having a plurality of wave translation devices included in tandem between said input and said output, each such wave translation device giving amplification between waves impressed upon it and waves output from it, and each such device having a feedback path for applying a part of its output wave to its input side in such manner as to increase the amplification, and an over-all feedback path from the output to the input of said system for increasing the resultant amplification of said tandem devices.

93. A wave translation system having an input and an output, means to apply waves to said input to be amplified, said system having a plurality of wave translation devices included in tandem between said input and said output, each such wave translation device giving amplification between waves impressed upon it and waves output from it, and each such device having a feedback path for applying a part of its output wave to its input side in such manner as to increase the amplification, and an over-all feedback path from the output to the input of said system for reducing the resultant amplification of said tandem devices, whereby the stability of the system is increased as regards variations in operation of said devices.

94. An active transducer having feedback, and means comprising a second active transducer for rendering the total feedback in said first mentioned transducer negative.

95. A feedback amplifier, and means comprising a second amplifier having feedback of one sign for causing the total feedback in said first mentioned amplifier to have the opposite sign.

96. An amplifier, and means therein comprising a positive feedback amplifier for producing in said first mentioned amplifier feedback in which $|\mu\beta|>>1$.

97. Two mutually exclusive amplifying paths, a common feedback path from the output of each to the input of each, and a transmission compensating network in said path.

98. An active transducer having separate feedback paths mutually conjugate.

99. A wave translating system having feedback means providing a closed loop transmission path in said system, the value of $|\mu\beta|$ being greater than unity and the values of $|\mu\beta|$ and Φ for said loop being so related that the feedback action in the system increases stability of transmission in the system as compared to the stability with no feedback action.

100. A wave amplifying system having a closed feedback loop wherein the value of $|\mu\beta|$ is greater than unity and the values of $|\mu\beta|$ and Φ are such that the feedback action in the system increases the gain of the system and at the same time increases the stability of the system.

101. A wave translating system having a feedback loop transmission path with the value of $|\mu\beta|$ greater than unity and the values of $|\mu\beta|$ and Φ such that feedback action in the loop decreases non-linearity of response of the system as compared to the non-linearity of response without feedback.

102. A closed feedback system comprising but two amplifier stages and one transformer in tandem and having $\mu\beta>>1$.

103. A wave amplifying system having an input portion and an output portion, means to impress waves on said input portion, means to amplify said waves and impress the amplified waves on said output portion, means associated with the output portion for utilizing the amplified waves, and means to reduce the gain of said amplifying means below the gain without feedback comprising means feeding back to the input a portion of the waves resulting from amplification of said impressed waves, said impressed and fedback waves producing resultant input waves, the ratio of amplitude of the waves so fed back to the amplitude of the resultant waves being substantially greater than unity for wave components of frequencies utilized by said means associated with the output.

104. A wave translating system including wave amplifying means having an input and an output, means to impress waves on the input, means deriving from the amplified waves wave components representing the amplified impressed waves and other wave components appearing in the amplifying means and not present in the impressed waves, and means to increase in the output of said system the ratio of amplified impressed waves to said other wave components comprising means to feed back to the input side of the amplifying means said derived components to reduce the gain of said amplifying means below the gain without feedback for said impressed waves and to oppose production in the output of said other wave components.

105. A non-oscillating wave amplifying system having a wave input portion and a wave output portion, said system without feedback being subject to instability of gain, and means feeding back waves from the output to the input portion of said system such that said system has greater gain stability than with zero feedback.

106. A non-oscillating wave translating system having a wave input portion and a wave output portion, said system without feedback being subject to variable phase shift, and means feeding back waves from the output to the input portion of said system such that said system has greater stability of phase shift than with zero feedback.

107. In a wave translating device having amplifying properties, an input and an output therefor, and means to increase gain stability of said device comprising means feeding back waves from the output to the input thereof in such manner as to reduce the gain of said device below the gain with zero feedback.

108. In a wave translating device having amplifying properties, an input and an output therefor, and means to increase stability of phase shift of said device, comprising means feeding back waves from the output to the input thereof in such manner as to reduce the gain of said device below the gain with zero feedback.

109. In a wave translating system, an amplifying portion, and means to increase gain stability of the system against variations in amplification in said portion, comprising means to feedback therein a portion of the translated waves reducing the gain of the system below the gain with zero feedback.

110. In a wave translating system, an amplifying portion, and means to increase stability of the phase shift through the system against variations in phase shift through said portion, comprising means to feed back therein a portion of the translated waves reducing the gain of the system below the gain with zero feedback.

111. A wave translating system comprising input and output portions and wave amplifying means therebetween, means to impress waves on said input portion for creating waves in said

output portion, and means responsive to waves in said output portion for applying waves to said input portion that form therein with said impressed waves resultant waves the ratio of whose amplitude to the amplitude of said applied waves is less than unity, said last-mentioned means comprising means adjustable for varying the gain of said system.

112. The combination with a transmission line for transmitting waves of a band of frequencies and having a phase distorting characteristic, of an amplifier at a point in said line having a gain reducing feedback circuit, said gain reducing feedback circuit including a network of similar phase distorting characteristic to that of said line.

113. The method of amplification with reduced distortion comprising providing in an amplifier excess gain over that required for the fundamental wave, and feeding back in the amplifier both fundamental and distortion products in such phase and amplitude that the fed-back fundamental waves reduce the gain of the amplifier for the fundamental waves to the said required value, and that the fed-back distortion components reduce the total distortion appearing in the output at a rate such that substantially a decibel reduction in distortion is produced per decibel reduction in gain.

114. The method of stabilizing the gain of an amplifier circuit comprising providing in the amplifier excess gain over that required for the fundamental wave, and feeding back fundamental and other output components in such amplitude and phase as to reduce the gain for the fundamental to said required value, thereby stabilizing the gain by at least the amount of the gain reduction.

115. In a multistage amplifier in which the amplitude of the signal increases from stage to stage, as a means of reducing distortion a feedback connection from the output side of the stage having the greatest distortion to a point on the input side of said stage, said feedback connection feeding back both fundamental and distortion components in such magnitude and phase as to reduce the gain of the amplifier for the waves being amplified and simultaneously reduce the distortion in a degree comparable with said reduction in gain.

116. In an amplifying system in which, in the absence of feedback, for a given amplitude of signal wave effective on the grid of a later stage a given amount of distortion is produced in the output of said stage, one or more earlier stages ahead of said later stage tending to increase the signal amplitude effective on the grid of that stage above said given amplitude, and a reverse feedback connection from the output of said later stage to an input element of a said earlier stage reducing the gain sufficiently to make the wave effective on the grid of said later stage as low as said given amplitude, whereby the distortion produced in said later stage is reduced to a level below said given amount.

117. In an amplifier having one or more early stages followed by a power stage, a reverse feedback connection around a portion of said amplifier including said power stage, said feedback connection feeding back both fundamental and distortion products in such phase and amplitude as to reduce the gain of the amplifier for the fundamental components and reduce the distortion by an amount comparable with the reduction in gain.

118. In a signal transmission system, a signal amplifying circuit portion having a distorting property, whereby the output signal waves from said portion are accompanied by distortion components, two circuit paths leading to the input of said circuit portion, one such path applying signal waves to said input and the other such path applying to said input a portion of the output waves containing said signal and distortion components in such phase that the signal waves applied from the latter path tend to reduce the amplitude of the output signal resulting from the signal waves applied to said input by said first path, and the distortion components so applied reduce the amplitude of the output distortion, and means for increasing the signal intensity in the first path to offset in part at least the tendency of the signal waves in the second path to reduce the amplitude of the output signal.

119. A wave translation system having an output portion and two input paths, said system producing amplification in waves applied to either input path, means impressing waves to be amplified on one of the input paths, said system in a non-oscillatory condition tending to produce variations in output that bear other than a constant linear relation to said impressed waves, and means to reduce such variations in output comprising means applying waves derived from the amplified output waves to the other of said two input paths in a direction to reduce the effective gain of the system for the waves impressed on said one input path below the gain without feedback and to oppose the production of said output variations.

120. In a signal receiving system, a multiplicity of electron tube amplification stages, and means coupling one amplification stage with a preceding amplification stage for feeding back energy to said preceding amplification stage in phase opposition to the incoming signaling energy in proportion to the amplitude of the incoming signaling energy, the amplitude of the fed back waves being large compared to that of the resultant of those waves and the incoming waves.

121. A radio broadcast receiver comprising an amplification system including a multiplicity of intercoupled electron tube amplification stages, means intercoupling a portion of the circuit of one electron tube amplification stage with the circuit of a preceding electron tube amplification stage for transferring current therebetween, and means for displacing the phase of the current thus transferred with respect to the phase of incoming signaling current in said preceding amplification stage for opposing increases and decreases in the incoming signaling current in such manner as to stabilize gain of the amplification system for maintaining the amount of energy in said amplification stages at a constant value.

122. An electrical system comprising an amplifier provided with an input and output circuit, means for impressing on the input circuit an alternating current voltage, an impedance in said output circuit, and means for impressing upon the input circuit, in phase opposition to said voltage, a fraction of the voltage developed across said impedance whose reciprocal has a predetermined small relation to the amplification power of said amplifier.

123. A method of operating an electron discharge tube amplifier to neutralize inherent distortion thereof consisting in impressing currents

upon the amplifier, detecting the amplified currents, and impressing a predetermined fraction of the detected currents which is small with respect to the amplification factor of the amplifier, upon the amplifier to neutralize said distortion, and maintaining said fraction constant in magnitude to render the amplified currents independent of the amplitude of the first named currents.

124. A method of operating an electron discharge tube amplifier to neutralize inherent distortion thereof consisting in impressing currents upon the amplifier, detecting the amplified currents, and impressing a predetermined fraction of the detected currents which is small with respect to the amplification factor of the amplifier, upon the amplifier to neutralize said distortion, said fraction being impressed upon the amplifier in phase opposition to the first named currents.

125. A method of operating an electron discharge tube amplifier to neutralize inherent distortion thereof consisting in impressing currents upon the amplifier, detecting the amplified currents and impressing a predetermined fraction of the detected currents upon the amplifier to neutralize said distortion and maintaining the product of the amplification power of said amplifier and the said fraction substantially greater than unity.

126. A method of operating an electron discharge tube amplifier to neutralize inherent distortion thereof consisting in impressing currents upon the amplifier, detecting the amplified currents and impressing a predetermined fraction of the detected currents upon the amplifier to neutralize said distortion and maintaining the magnitude of said fraction at such a value that amplified currents are equal to the product of the reciprocal of the fraction and said first named currents.

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